Material Point Method

Theory and Applications

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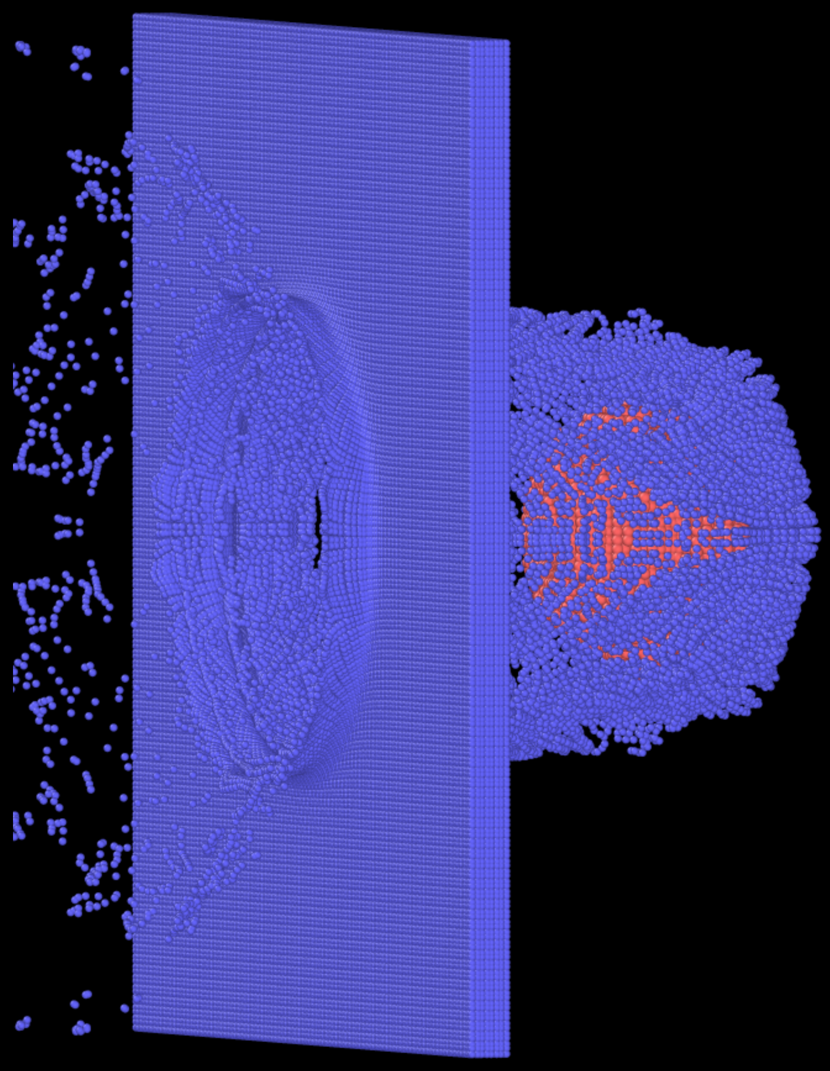

Collaborators and funding agencies

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Deakin University

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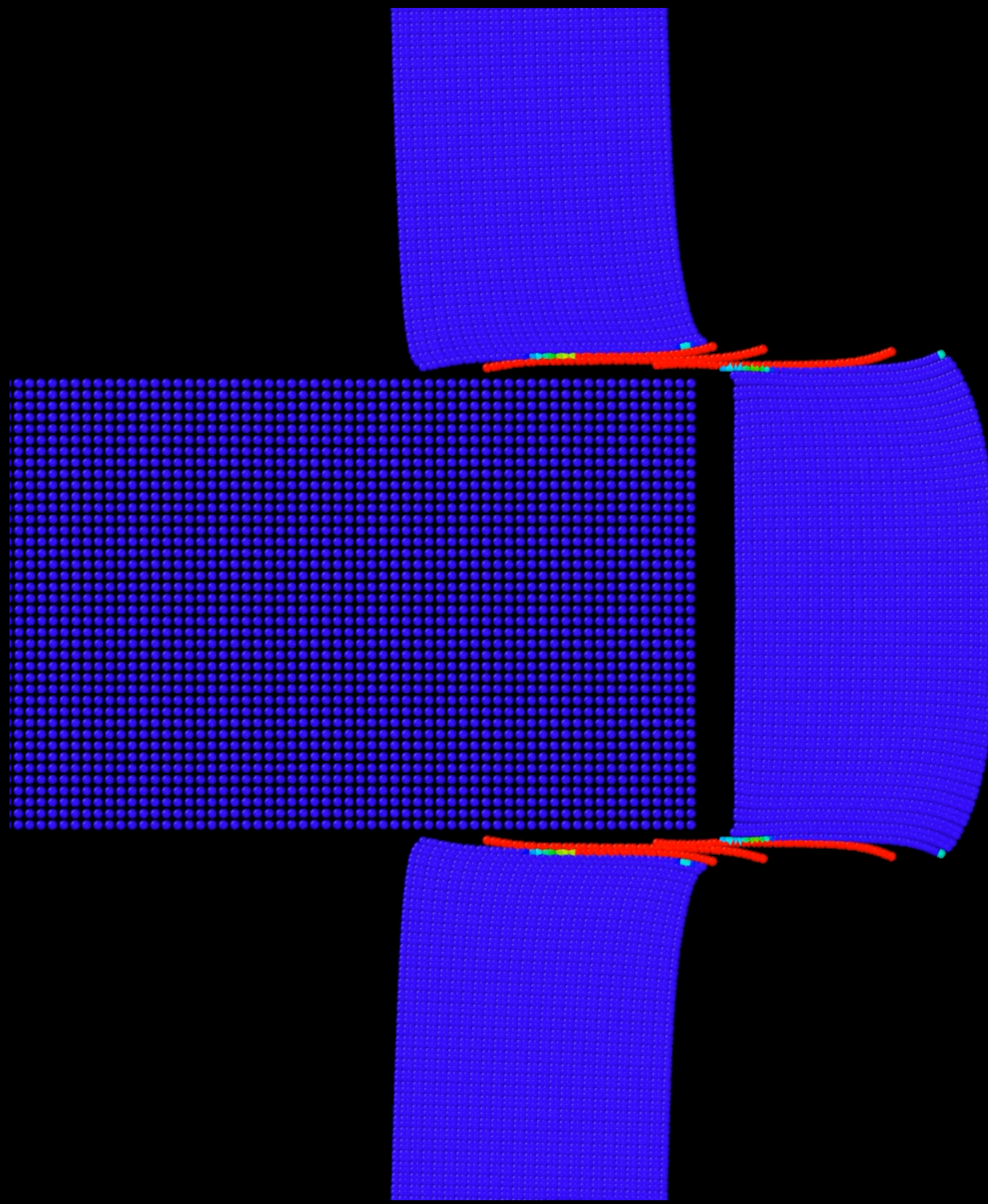
Melbourne University

Tushar Mandal
PhD candidate

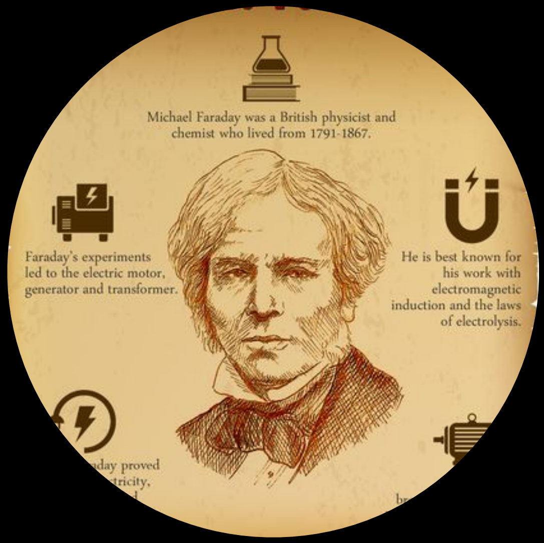
Australian Government

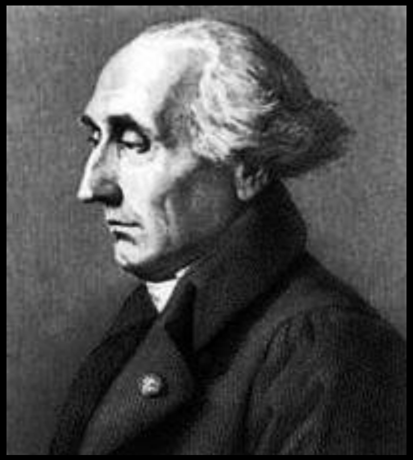
Australian Research Council

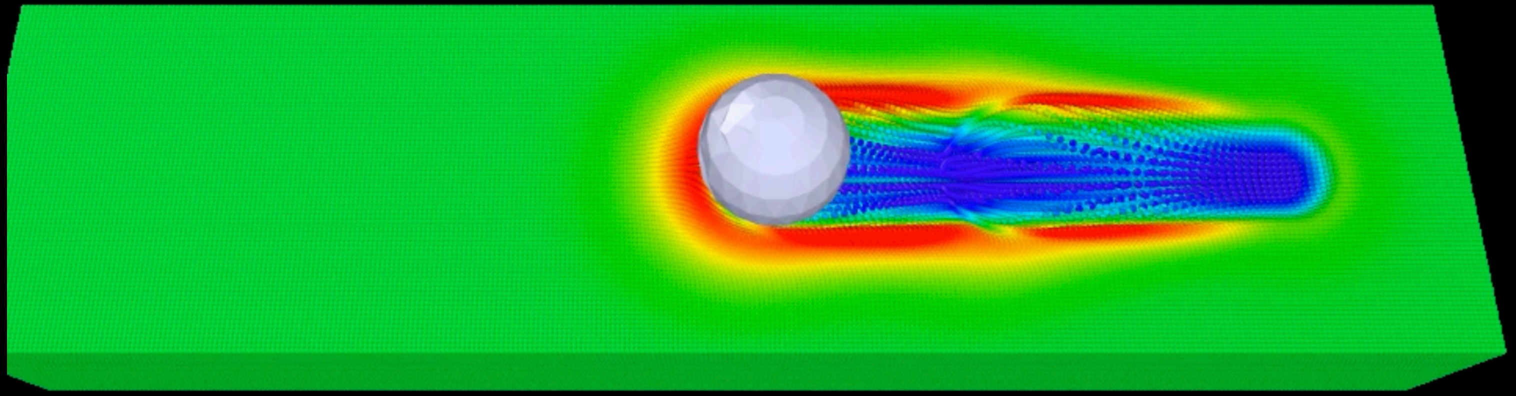
The problem: deformation/fracture of solids



Deformation/fracture of solids: approaches



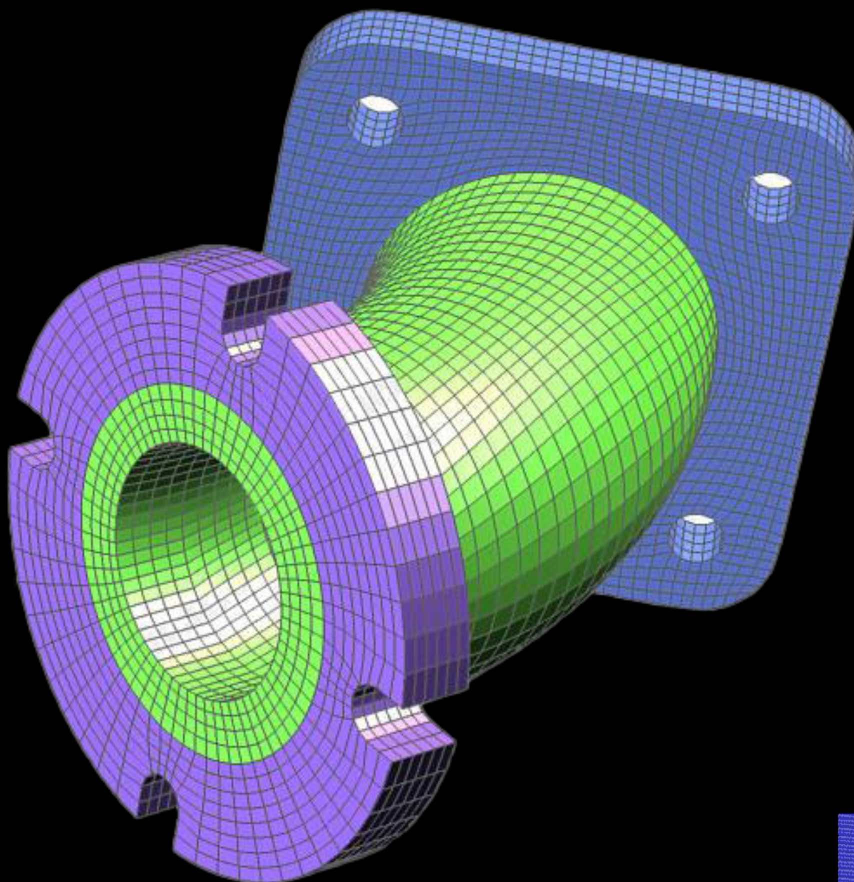




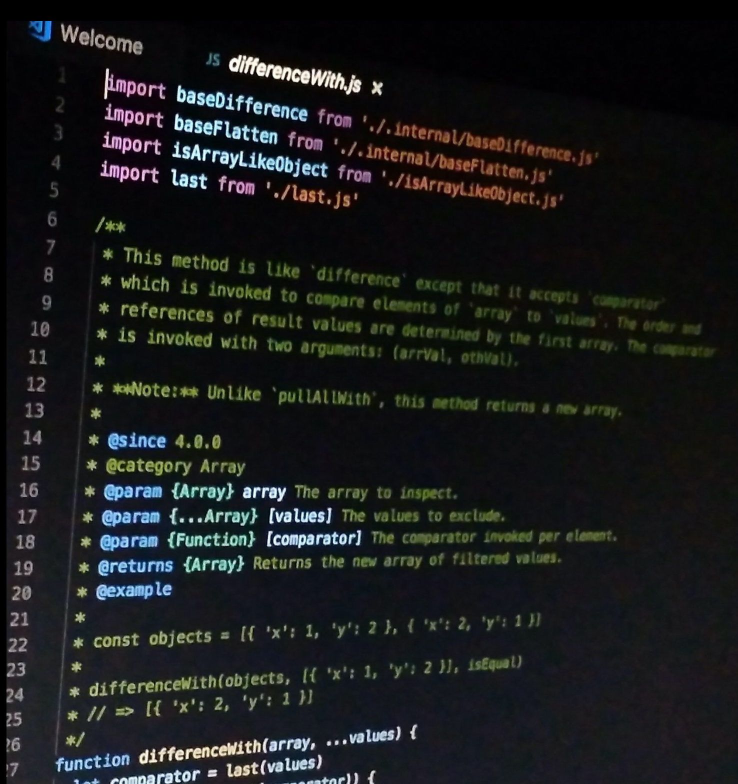
What computational engineer/scientist do

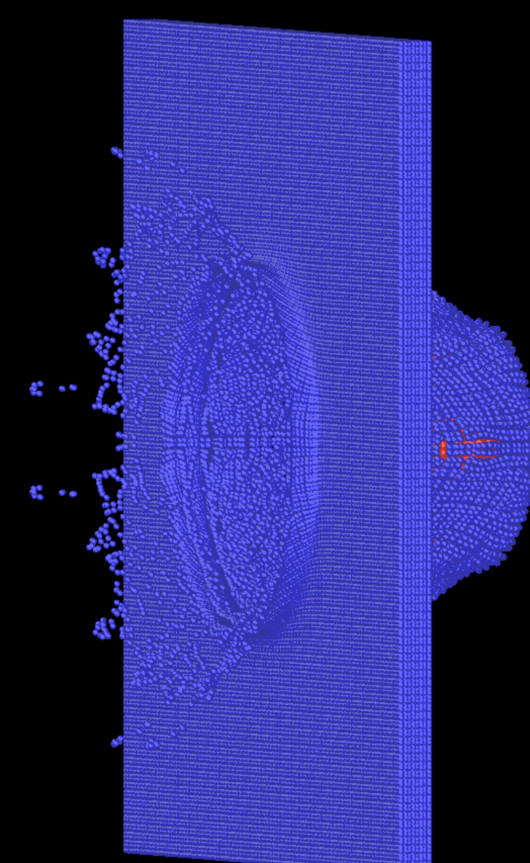
Balance equations
Constitutive models

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = f$$

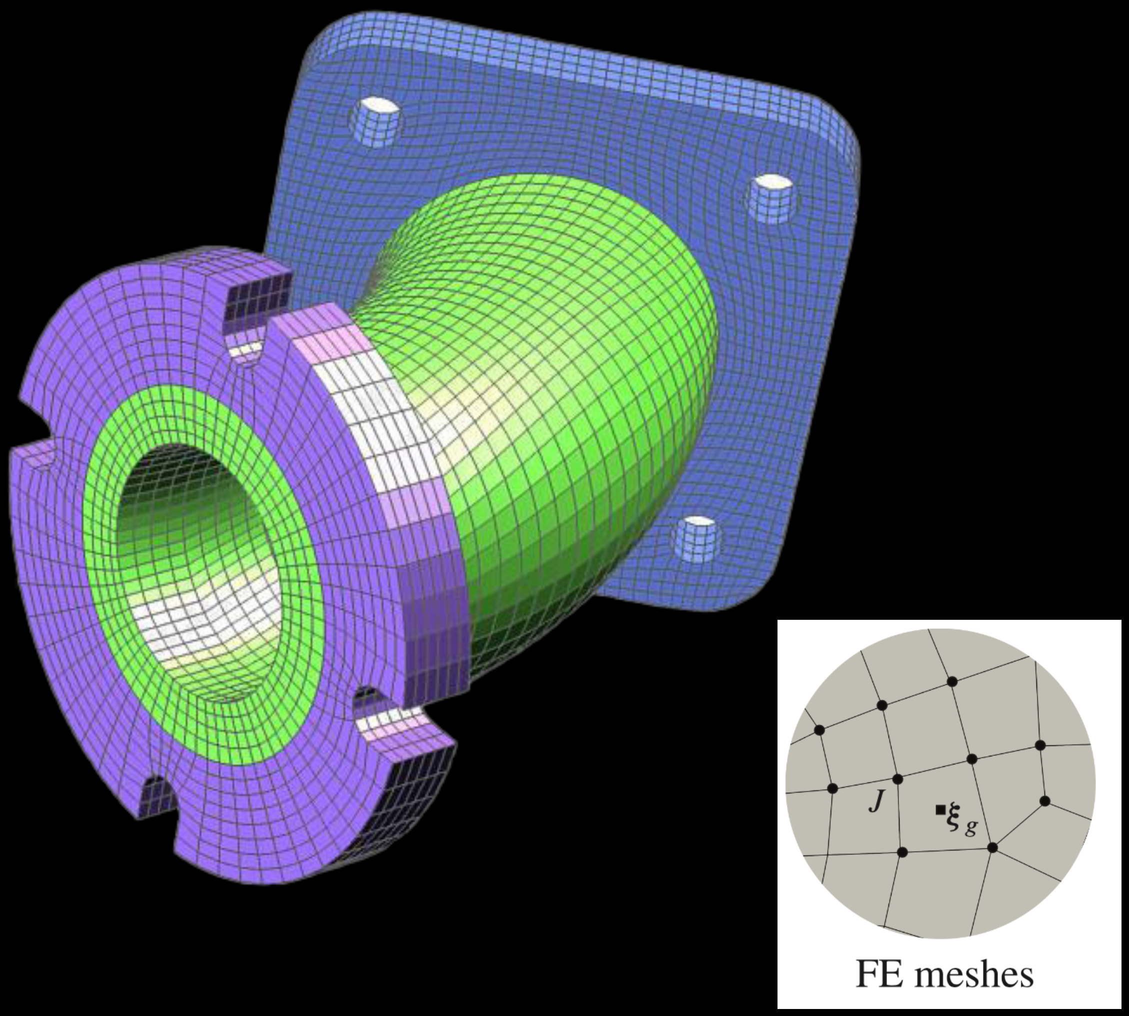


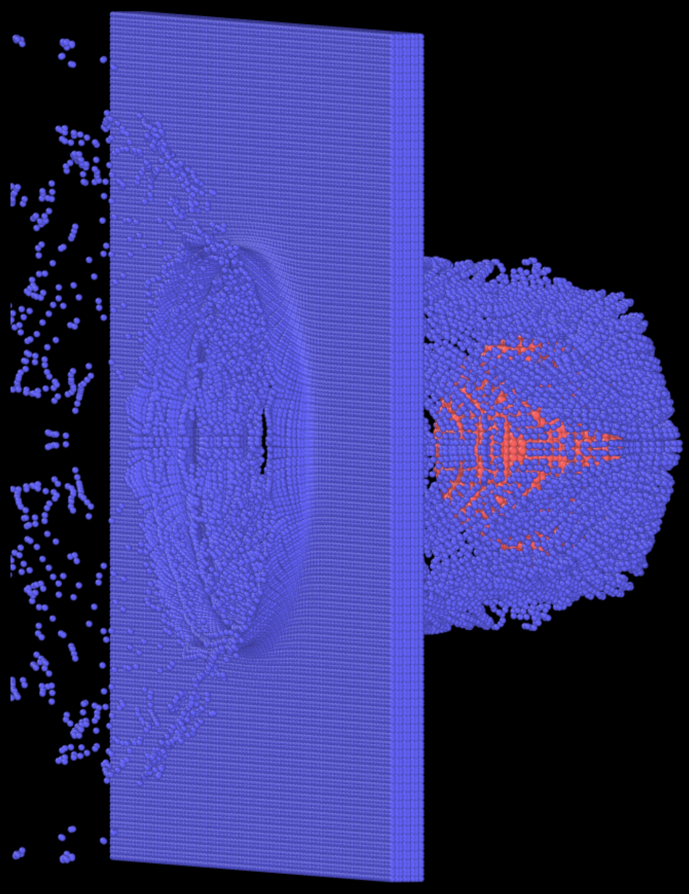
$$3x + 2y - z = 1$$
$$1x - 2y + 4z = 2$$
$$4x + 3y - 2z = 5$$





Mesh based and mesh-free methods

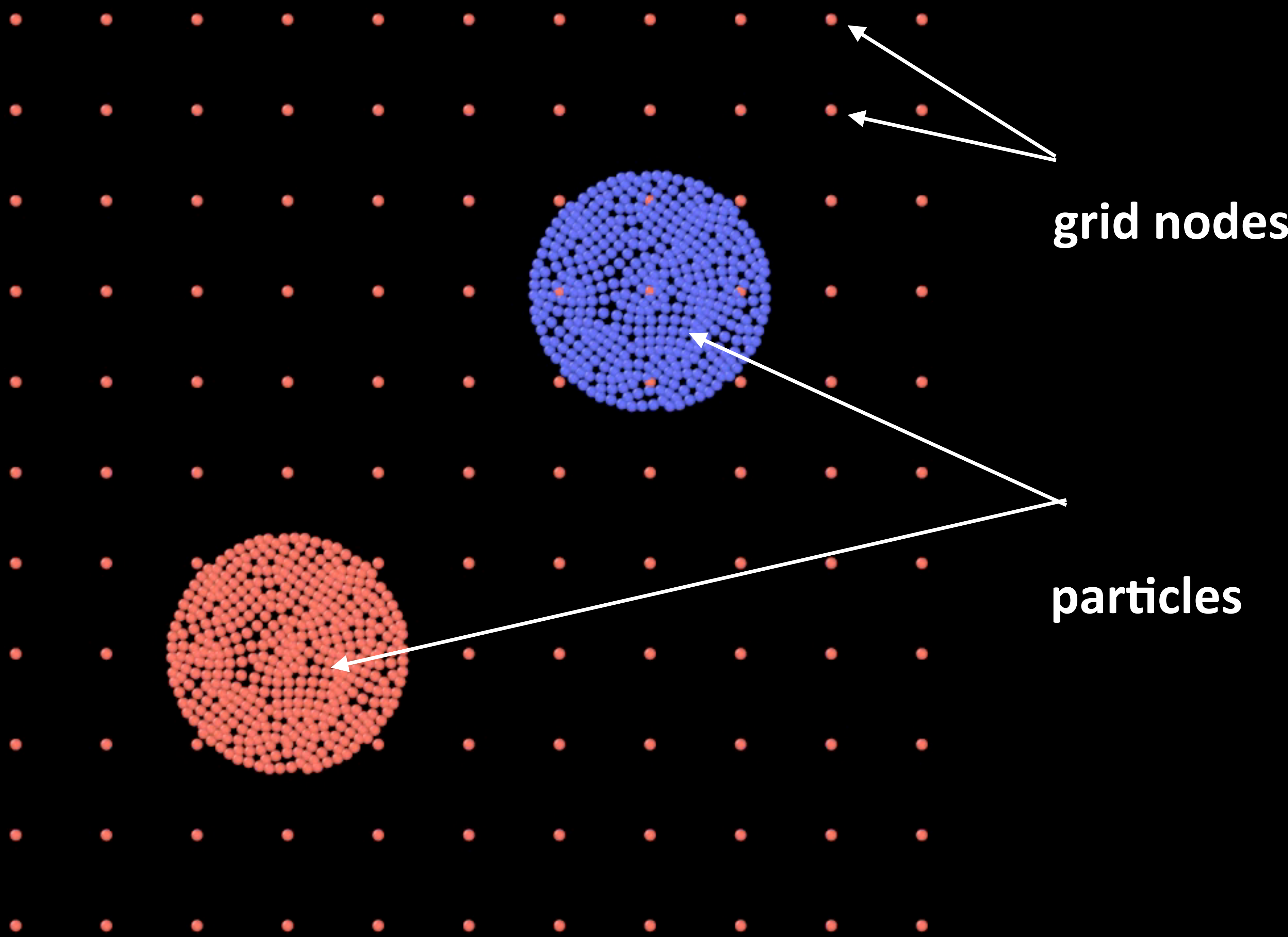




Basic concept: a mesh/grid
FDM, FVM, FEM
Efficient, accurate
FEM: prone to mesh distortion

Basic concept: points/particles
EFG, SPH, PFEM, MPM, …
Mesh distortion free
Inaccurate geometry

Material Point Method (MPM): particles over a fixed grid

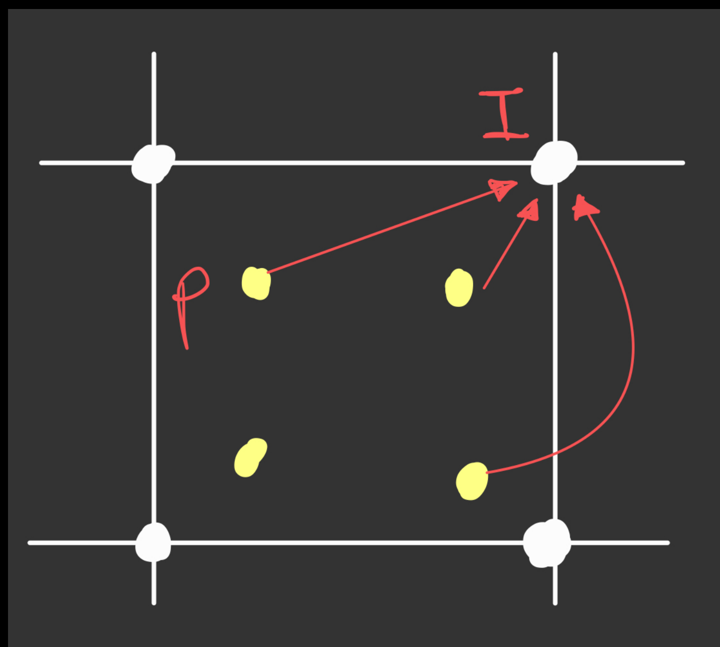


position
volume
momenta
stress
temperature
…

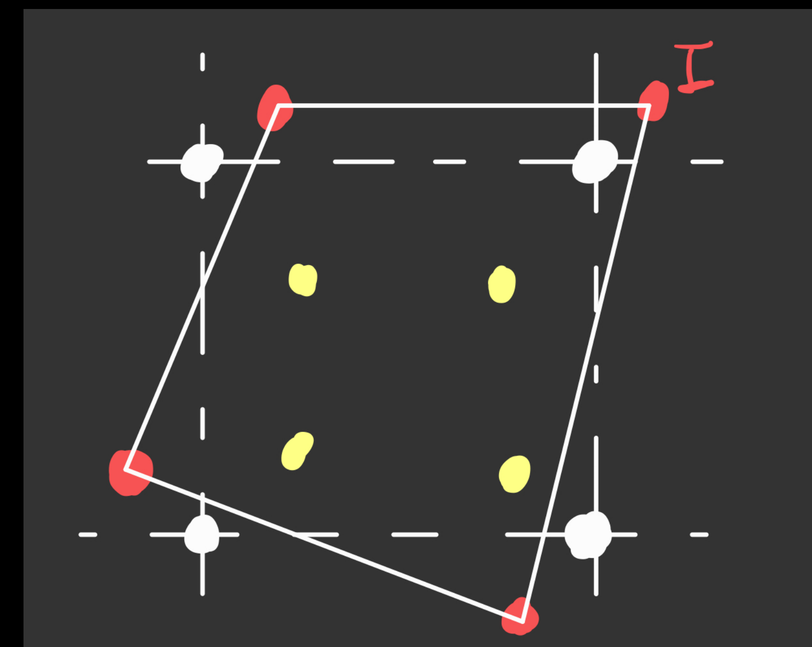
Sulsky et al, CMAME, 1994

Material Point Method: algorithm (explicit, ULMPM)

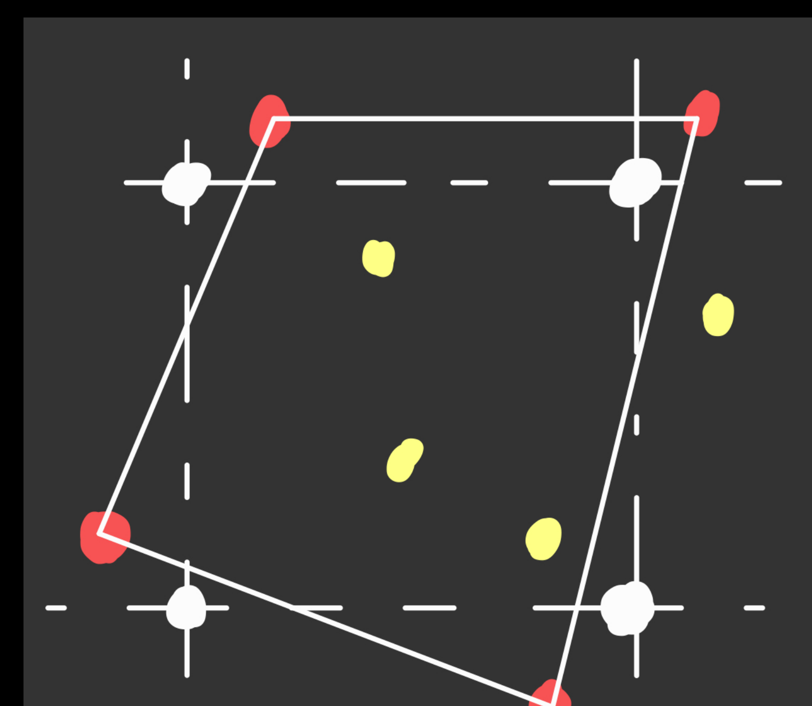
1 P2G



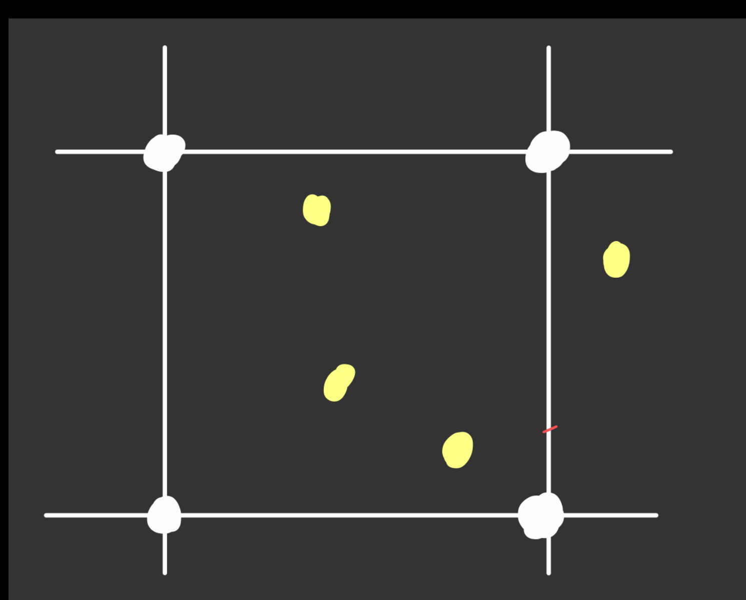
2 Grid update



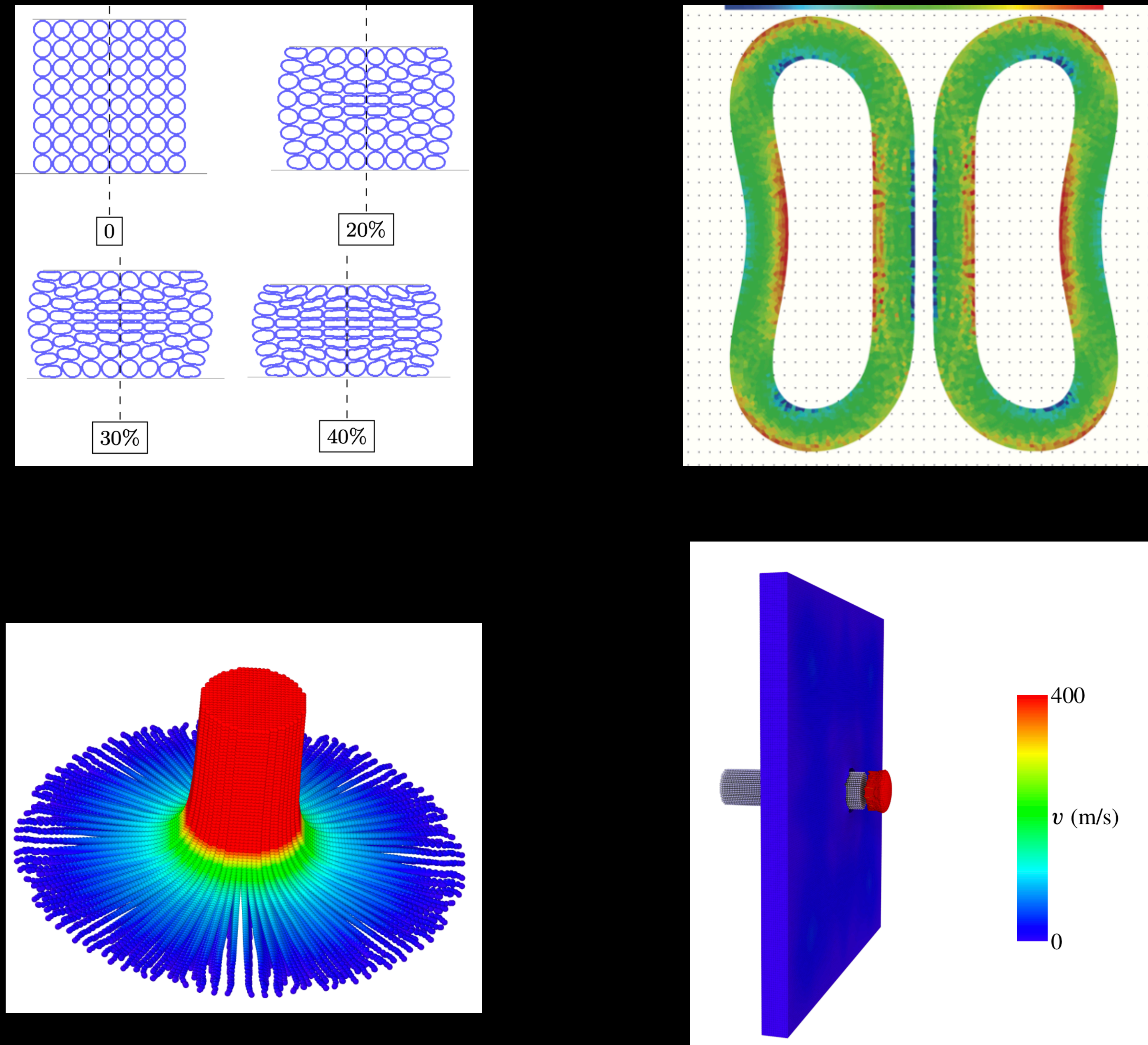
3 G2P



4 Grid reset



MPM really can simulate complex problems


0
20%
30%
40%
400
v (m/s)
0

MPM: Example 1

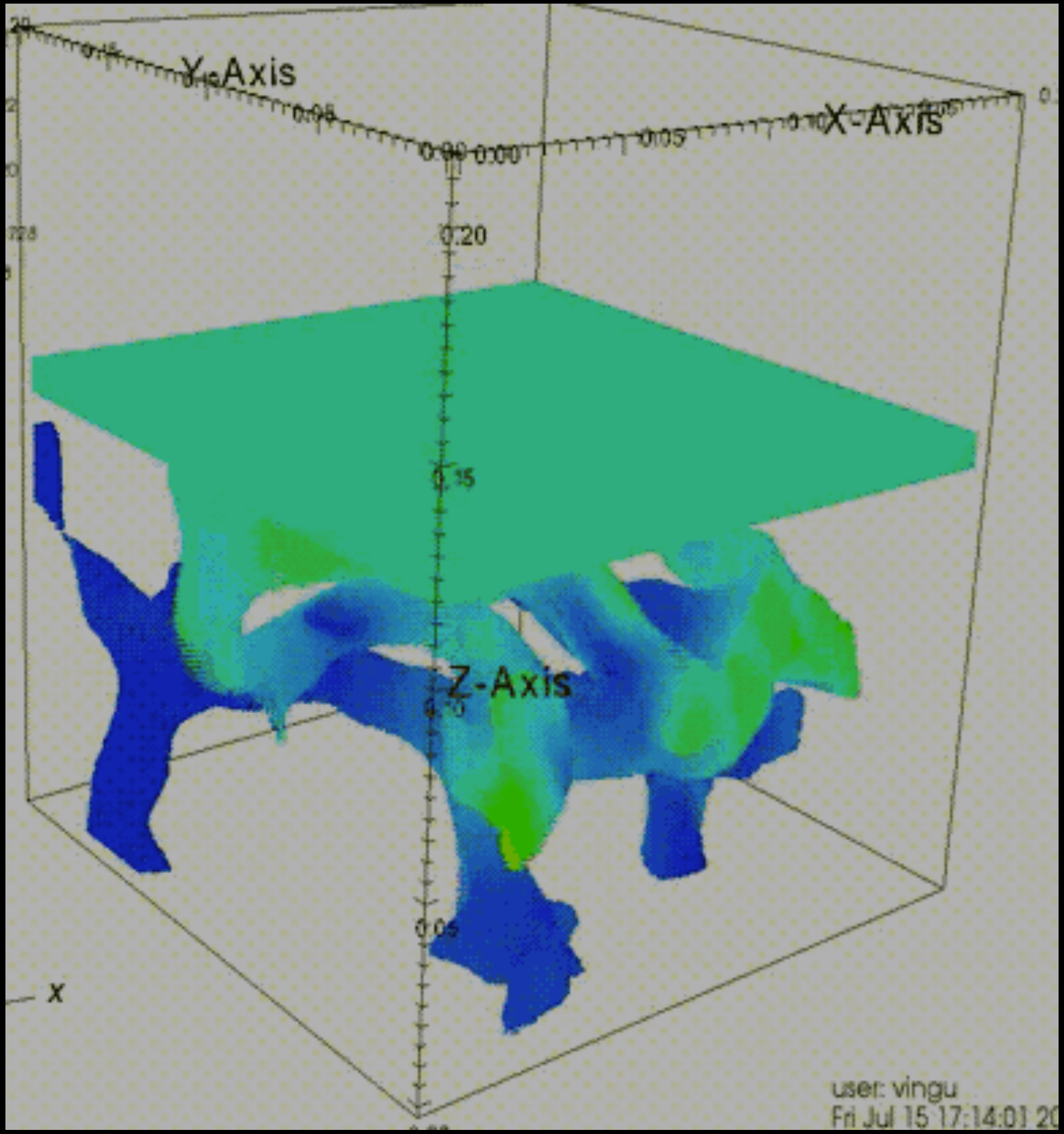
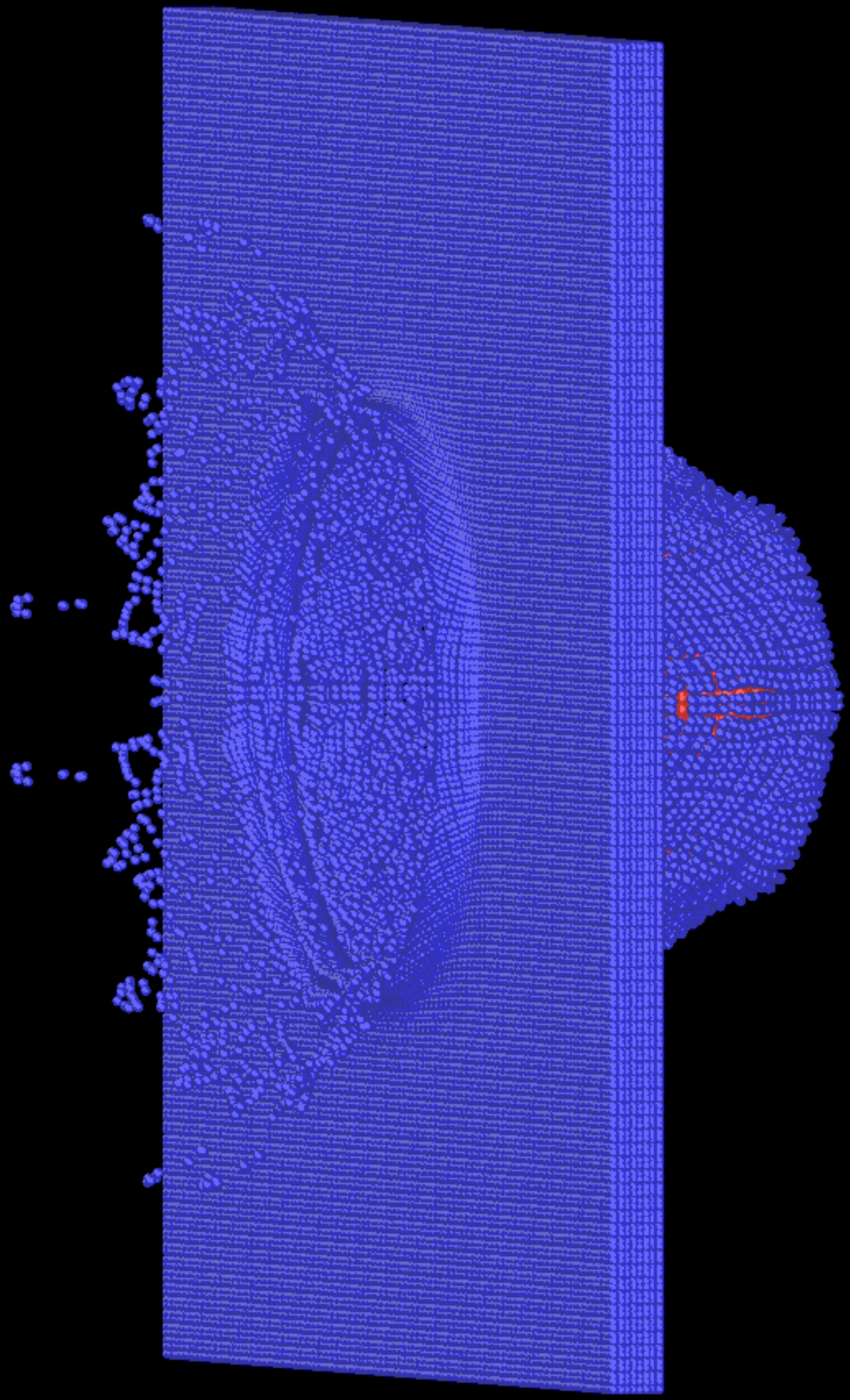


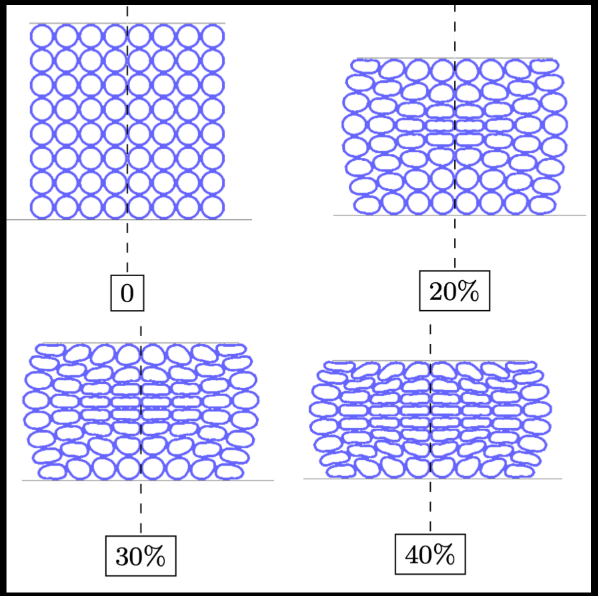
Image based analysis
Foamed materials: network of struts
Very large deformation
Lots of self contacts

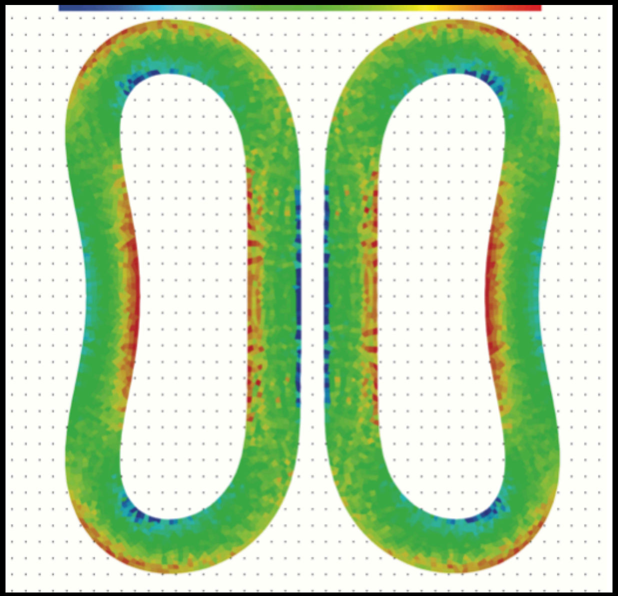
Sulsky et al, CMAME, 1994
Bardenhangen et al, JMPS, 2005

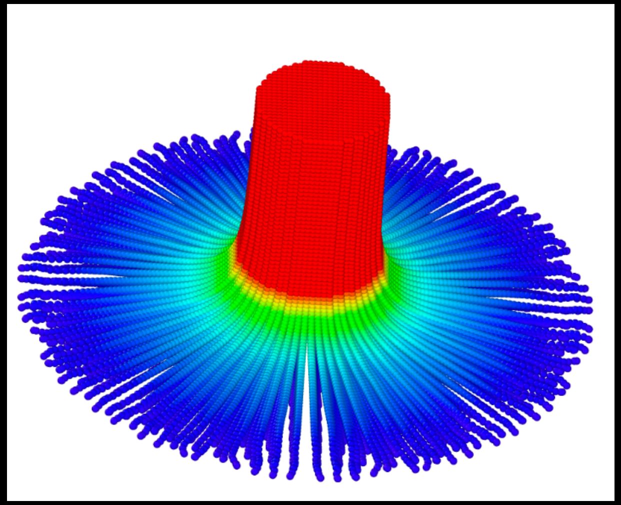
MPM: example 2

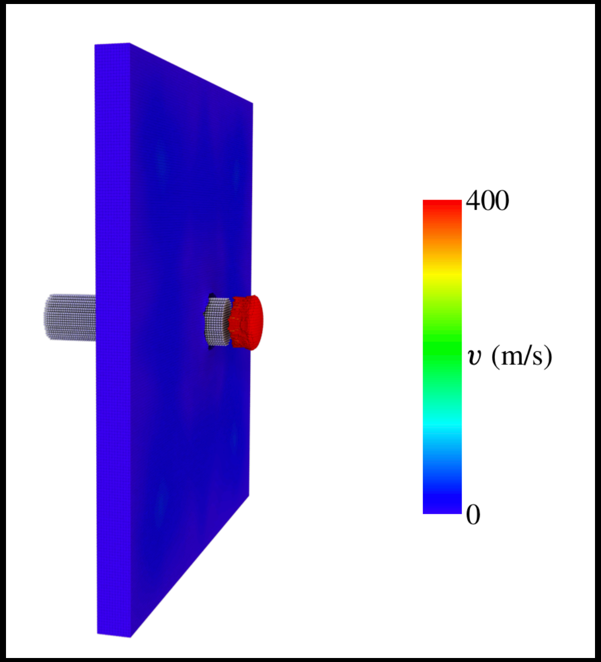


Hyper-velocity impact
Steel plate: Johnson-Cook model
Massive deformation
Karamelo (in-house C++ MPI)







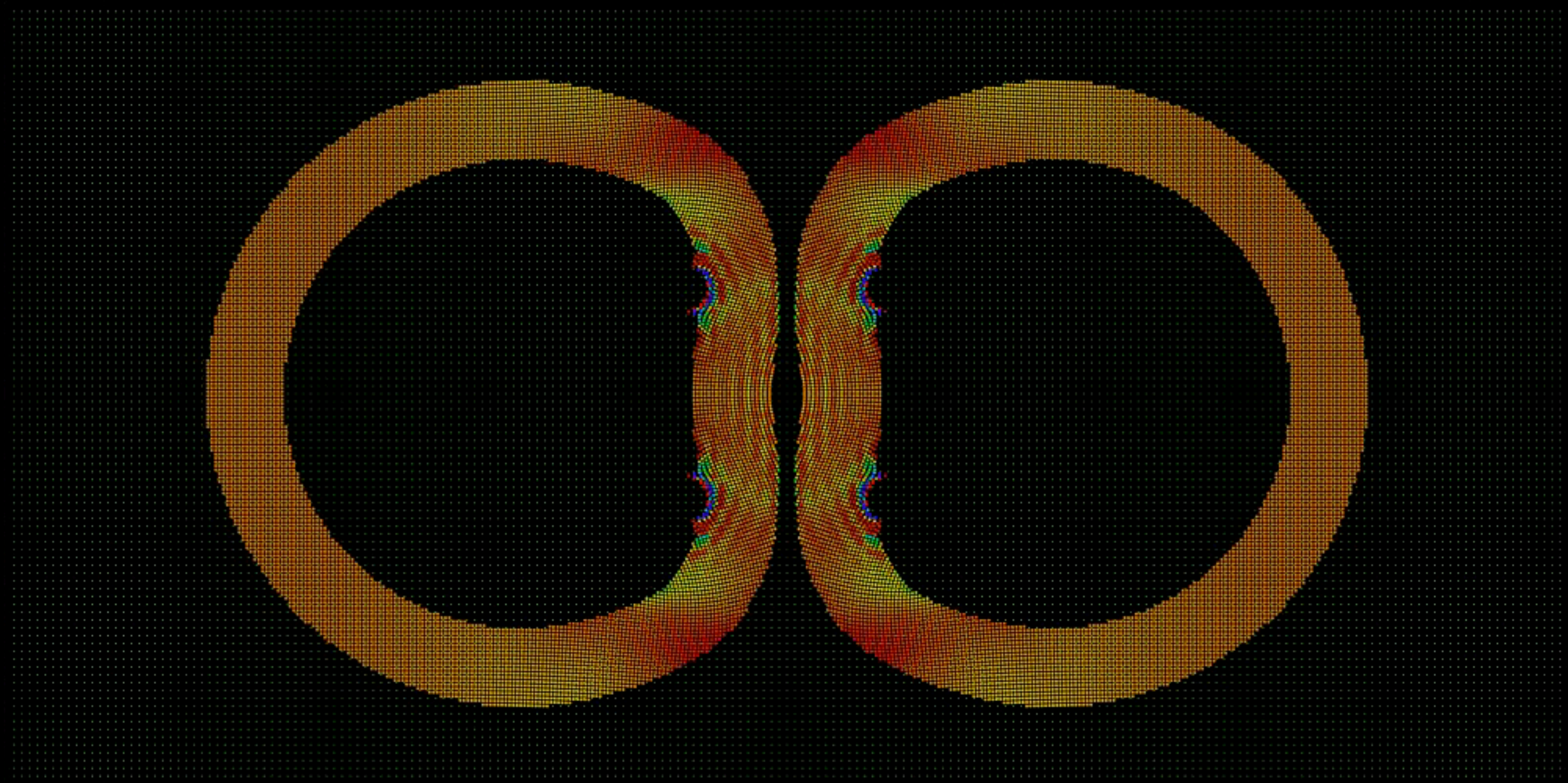


no mesh generation
simple basis functions
large deformation
automatic no-slip contact
multiple contacts
simple BC treatment
simple implementation

MPM really can simulate complex problems

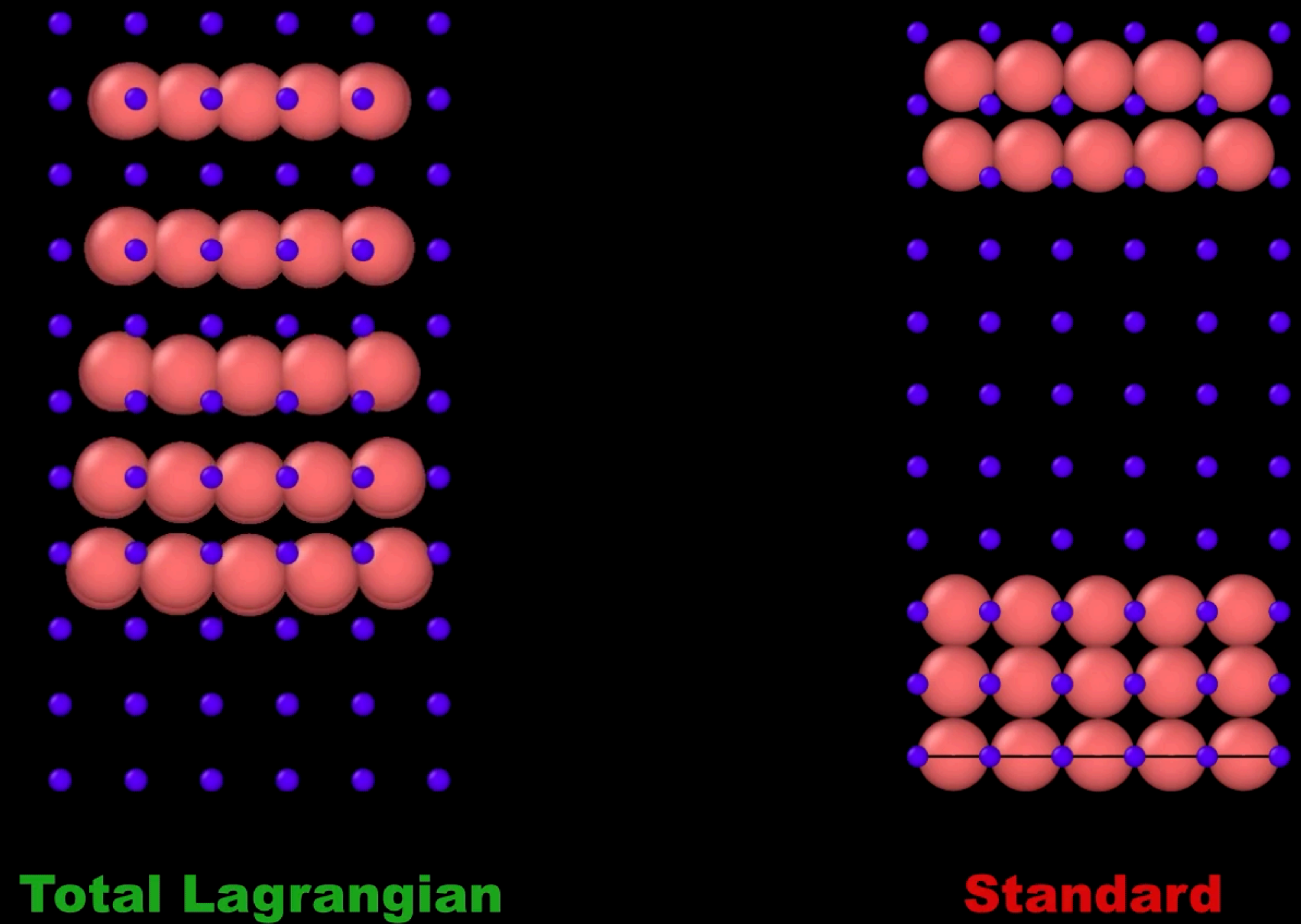
BUT

(UL)MPM: issues



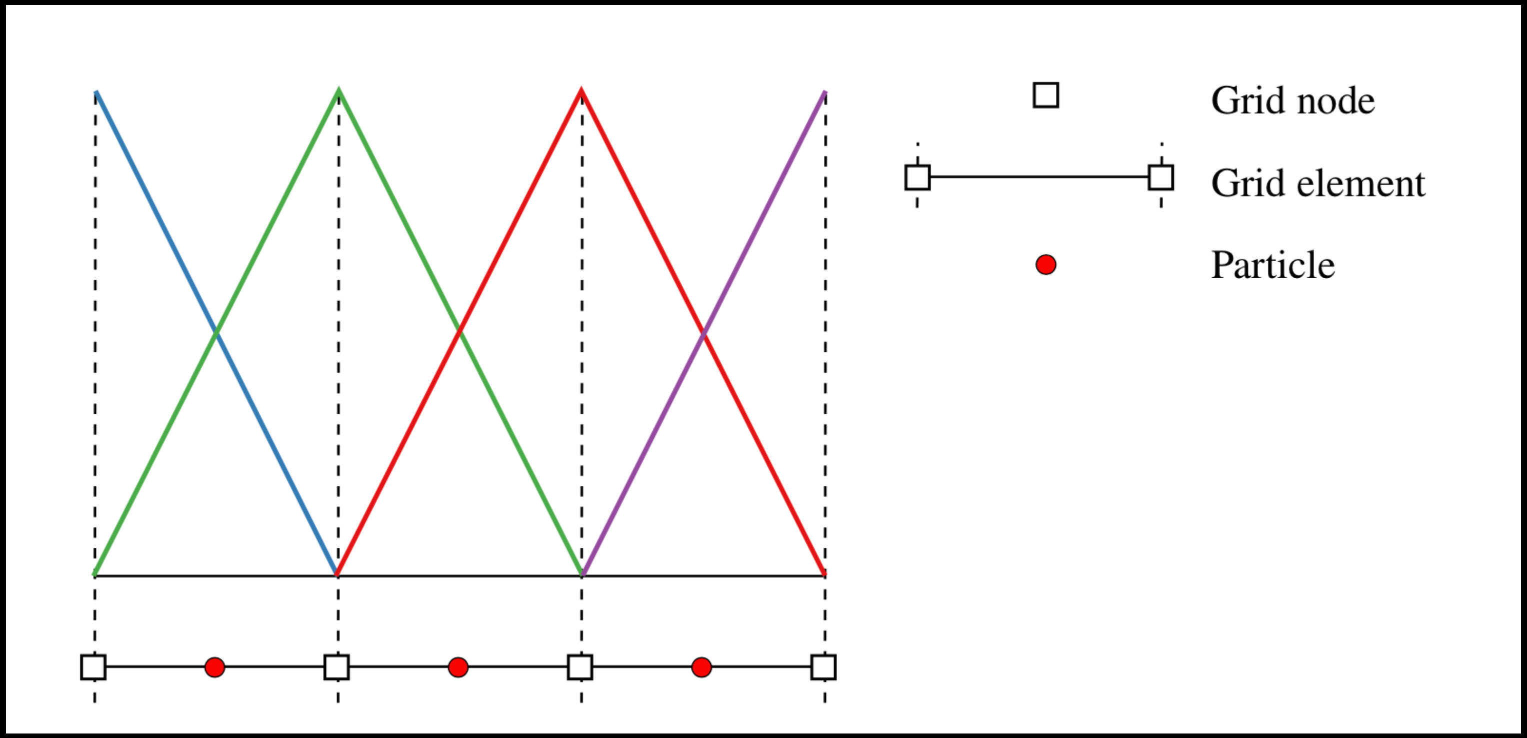
cell-crossing instability

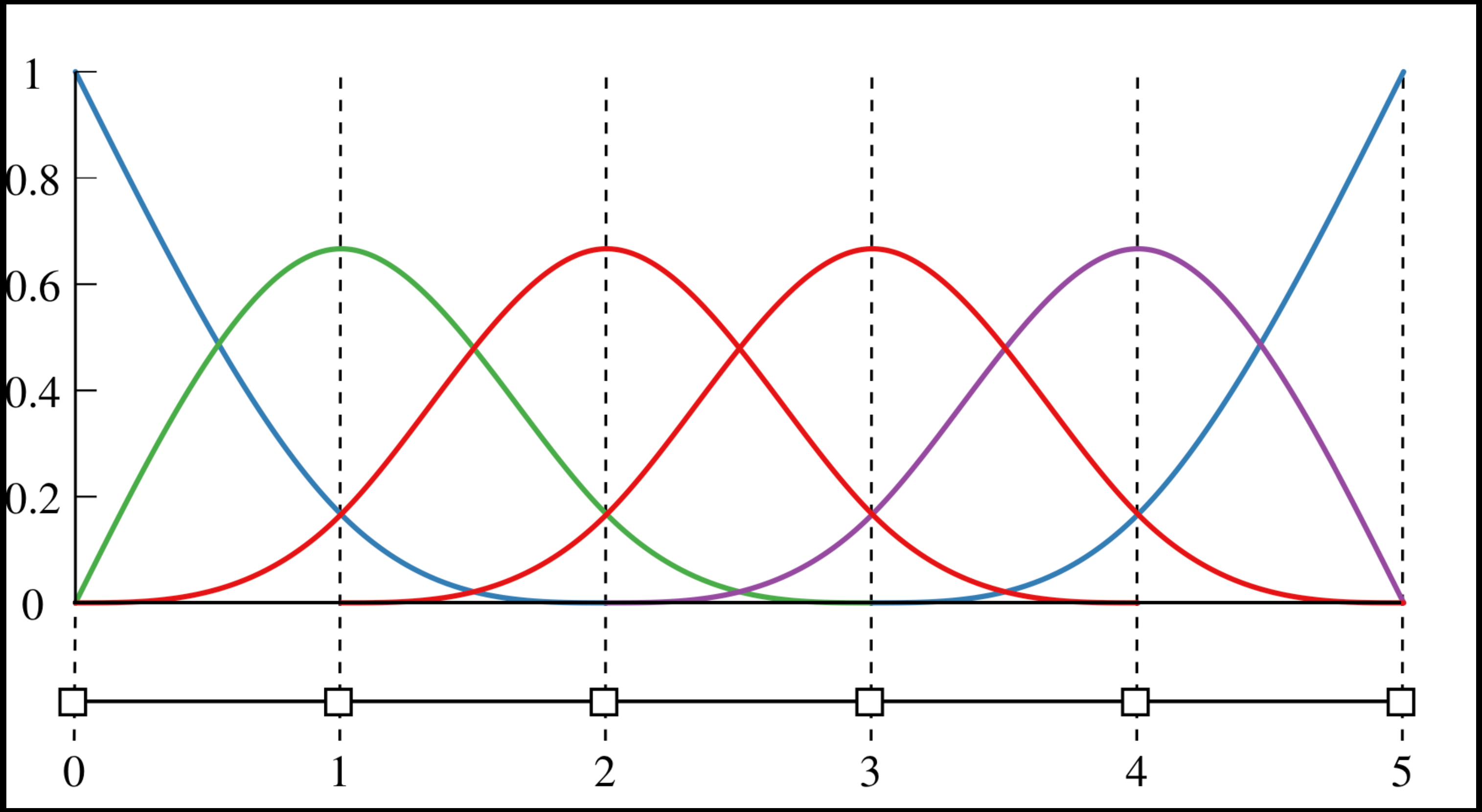
(UL)MPM: issues



numerical fracture

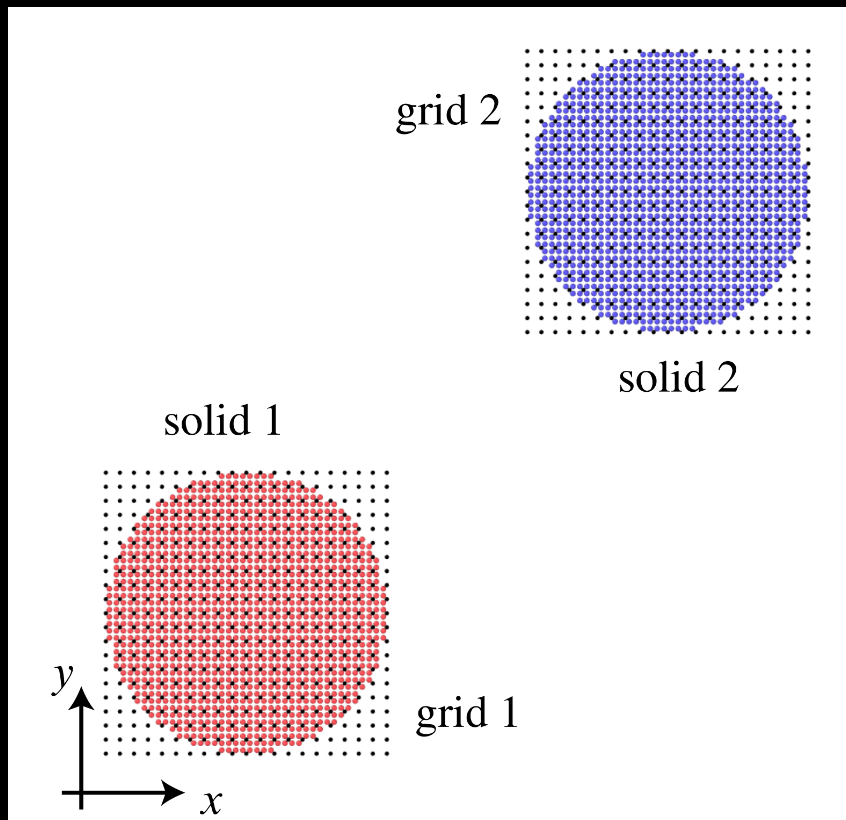
ULMPM: issues and *partial* solutions



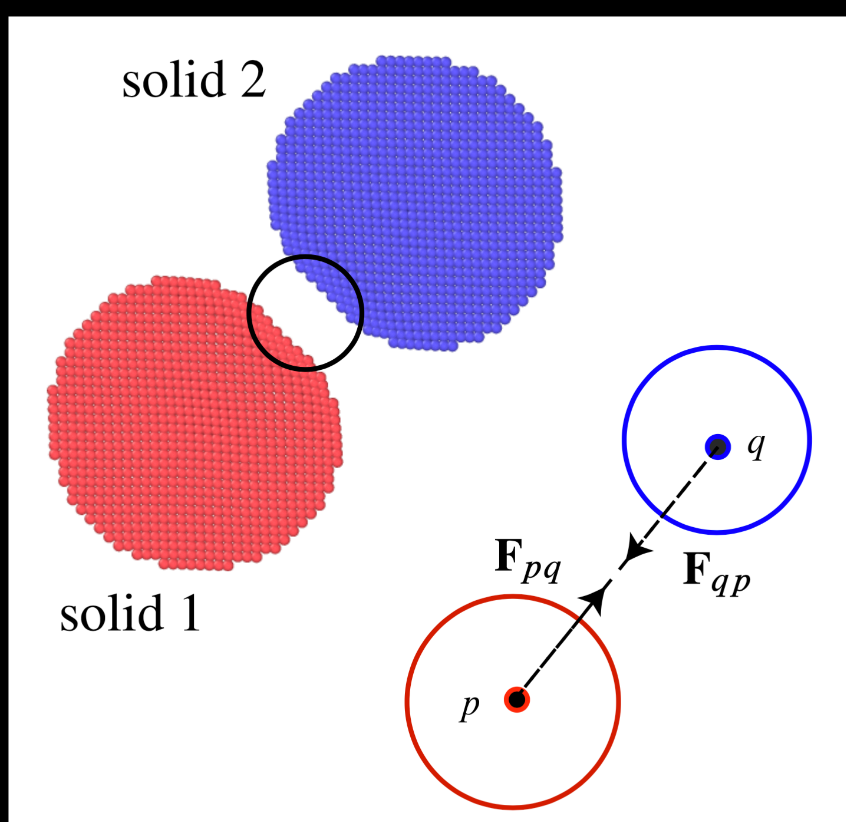


We present two solutions to solve issues of ULMPM

Total Lagrangian MPM (TLMPM): initial configuration



use the initial undeformed configuration
no cell-crossing instability
no numerical fracture
most accurate quadrature
no inherent no-slip contact
small time step for compression

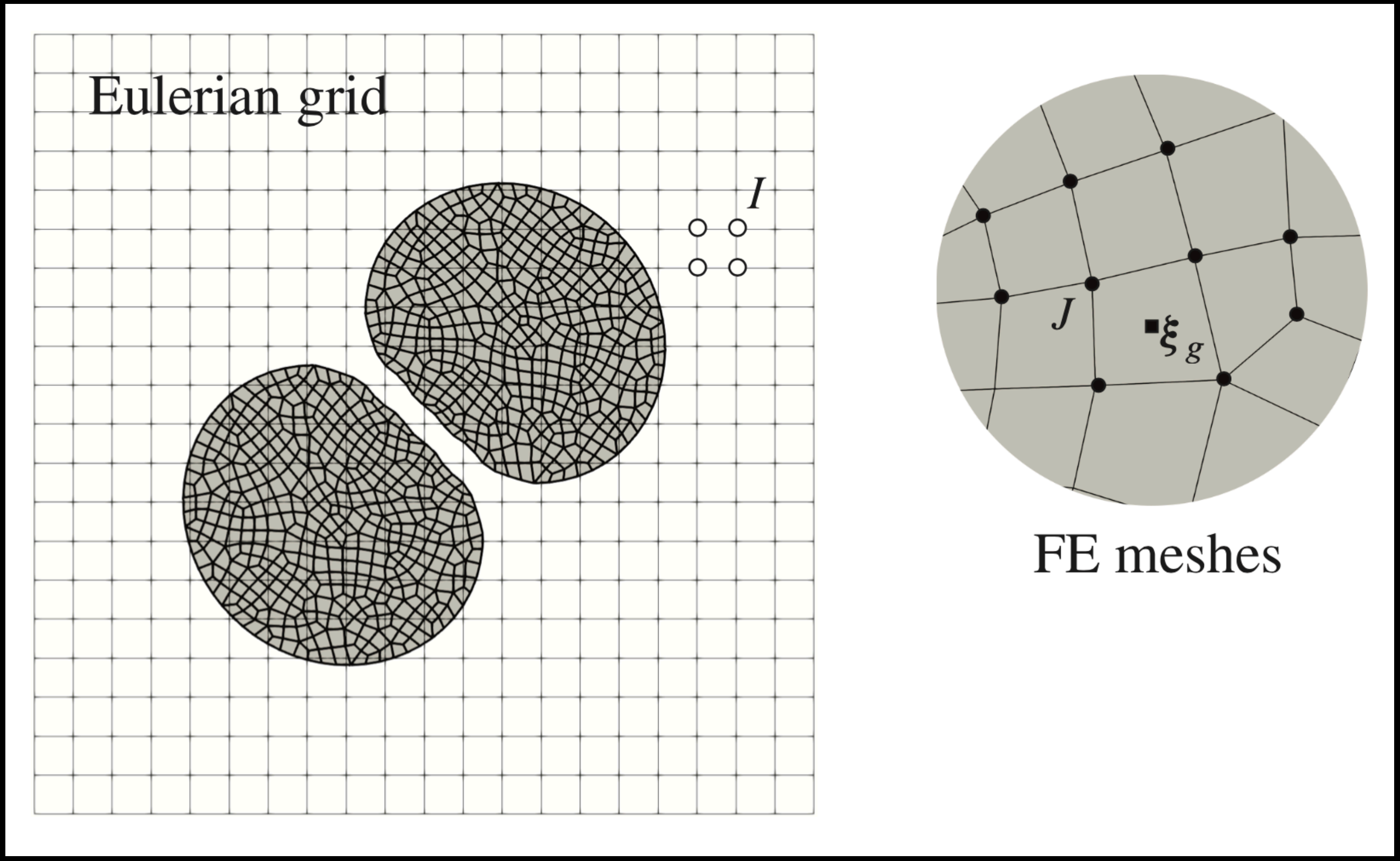


$$\delta^t = R_p + R_q - ||\mathbf{x}_q^t - \mathbf{x}_p^t||$$

$$\mathbf{F}_{pq} = \frac{1}{\Delta t^2}\frac{m_p m_q}{m_p + m_q}\left(1 - \frac{R_p + R_q}{\|\mathbf{x}_{pq}^t\|}\right)\mathbf{x}_{pq}^t$$

A. de Vaucorbeil, V. P. Nguyen, and C. R. Hutchinson. A total-lagrangian material point method for solid mechanics problems involving large deformations. CMAME, 2020.
A. de Vaucorbeil and V.P. Nguyen. Modeling contacts with a total lagrangian matertial point method. CMAME 2021.

Generalised Particle in Cell (GPIC): mesh again



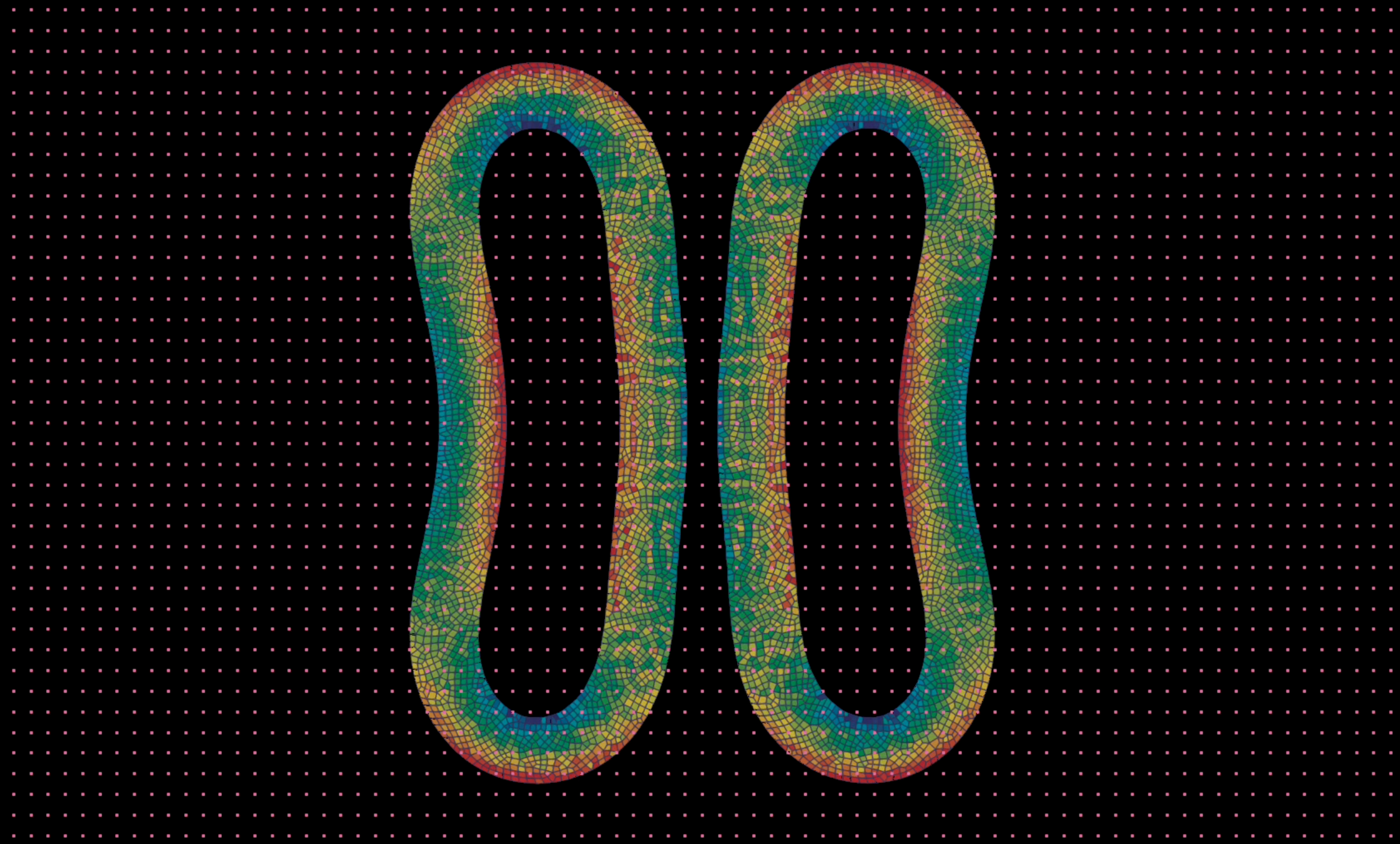
solids are meshed
initial configuration: forces
current configuration: contact

no cell crossing issue
no numerical fracture
accurate quadrature
accurate boundary
easy boundary conditions
easy material interfaces

fragmentation?
extreme deformation?

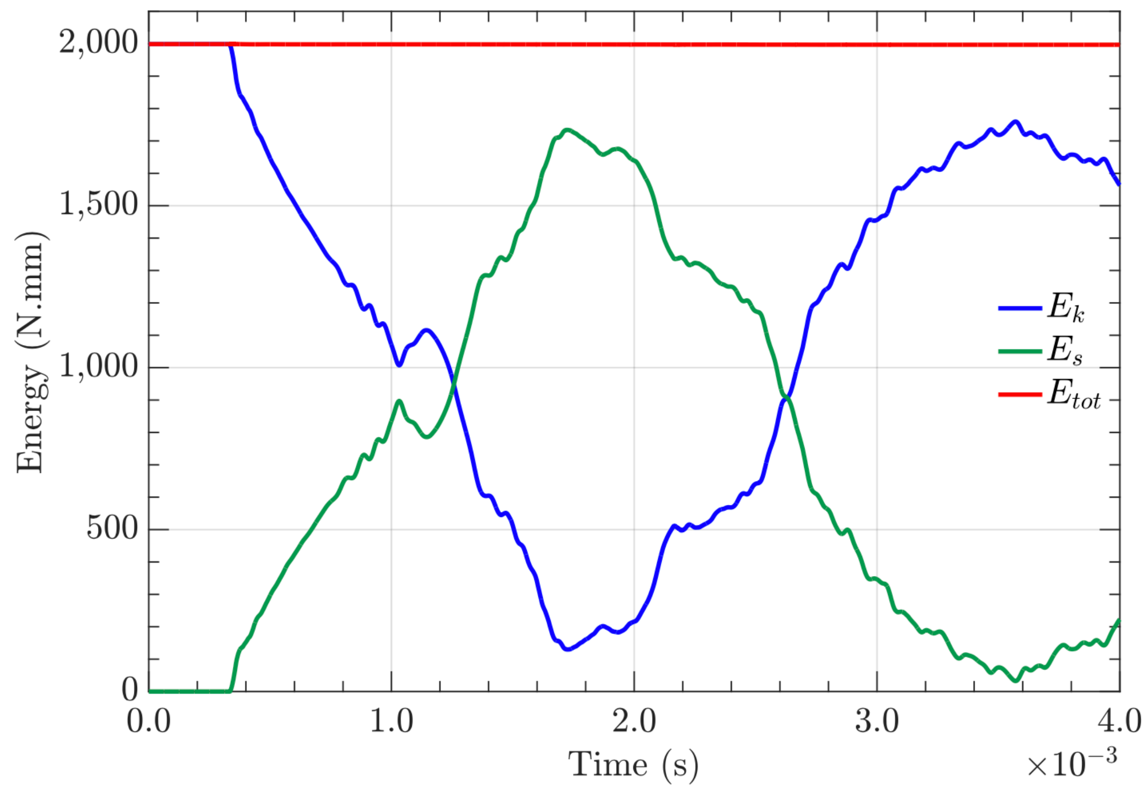
1. A. Sadeghirad, R. M. Brannon, and J. Burghardt. A convected particle domain interpolation technique to extend applicability of the material point method for problems involving massive deformations. International Journal for Numerical Methods in Engineering, 86(12):1435–1456, 2011.
2. V. P. Nguyen, C. T. Nguyen, T. Rabczuk, and S. Natarajan. On a family of convected particle domain interpolations in the material point method. Finite Elements in Analysis and Design, 126:50–64, 2017.
3. V.P. Nguyen, A. de Vaucorbeil, C. Nguyen-Thanh, and T. Mandal. A generalized particle in cell method for explicit solid dynamics. Computer Methods in Applied Mechanics and Engineering, 360:112783, 2020.

Generalised Particle in Cell (GPIC): example 1

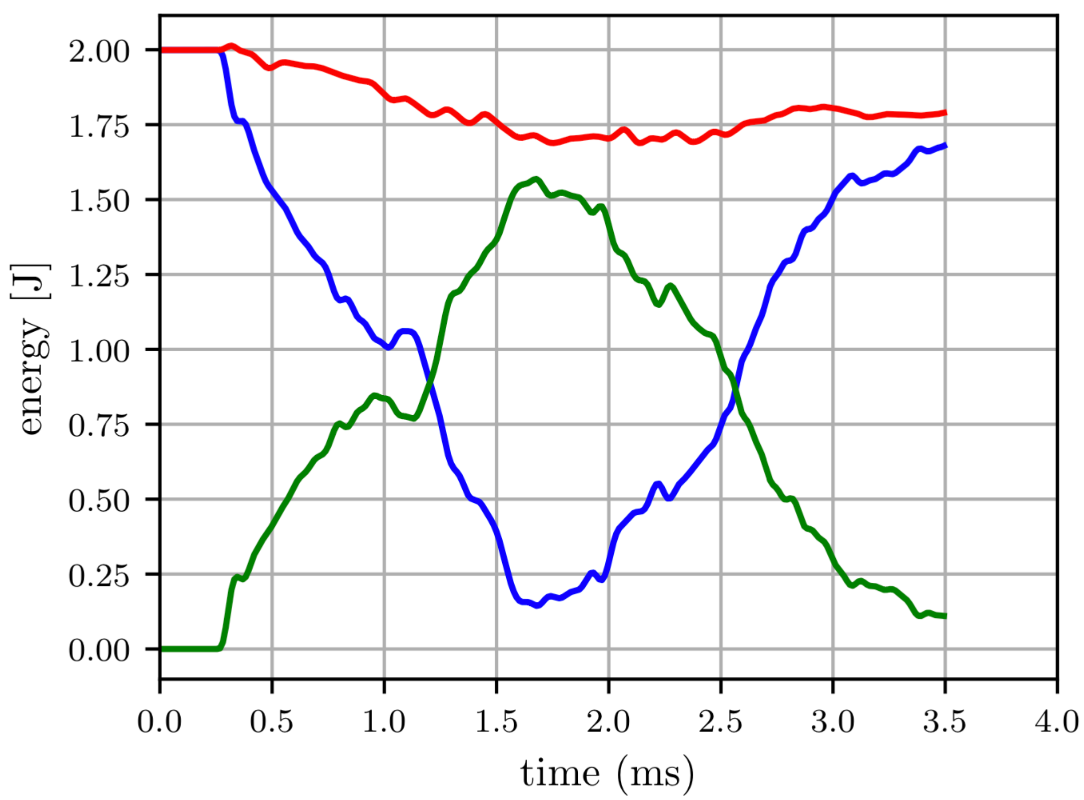


Generalised Particle in Cell (GPIC): example 1

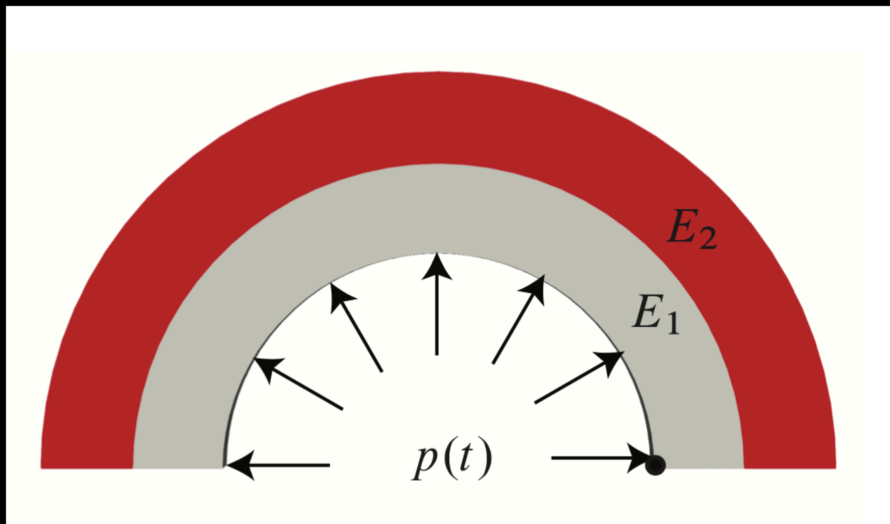
ABAQUS

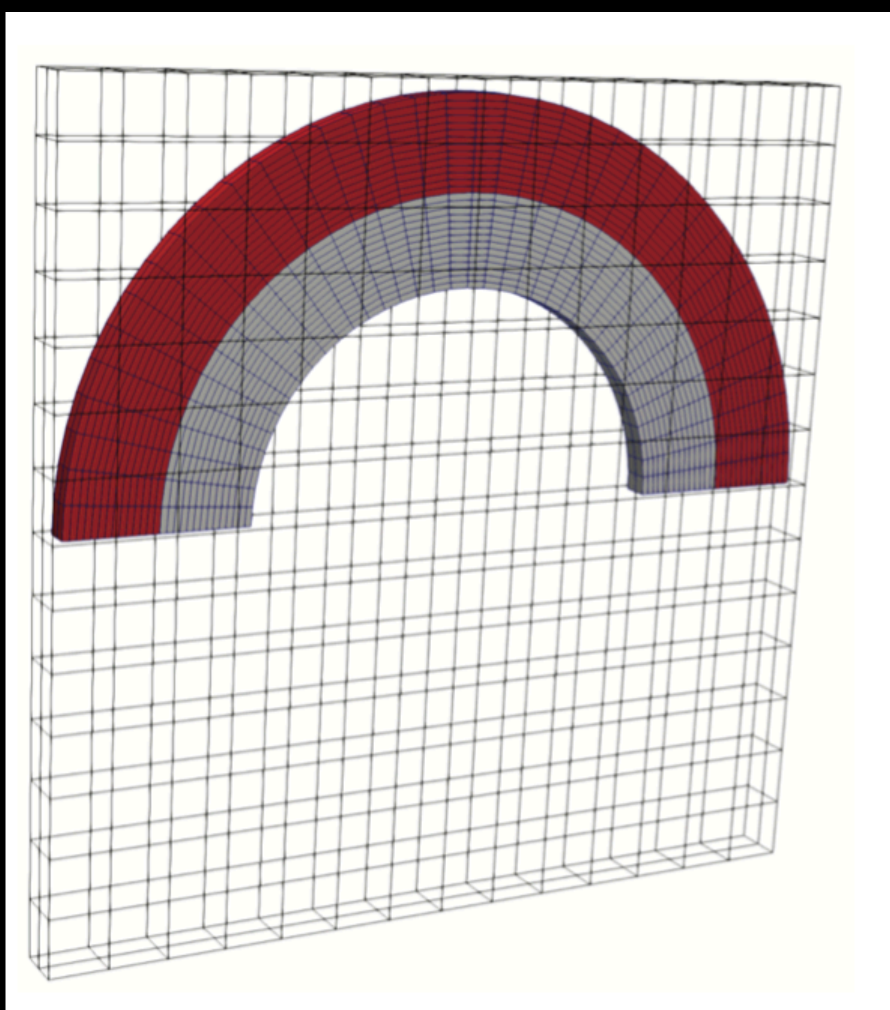


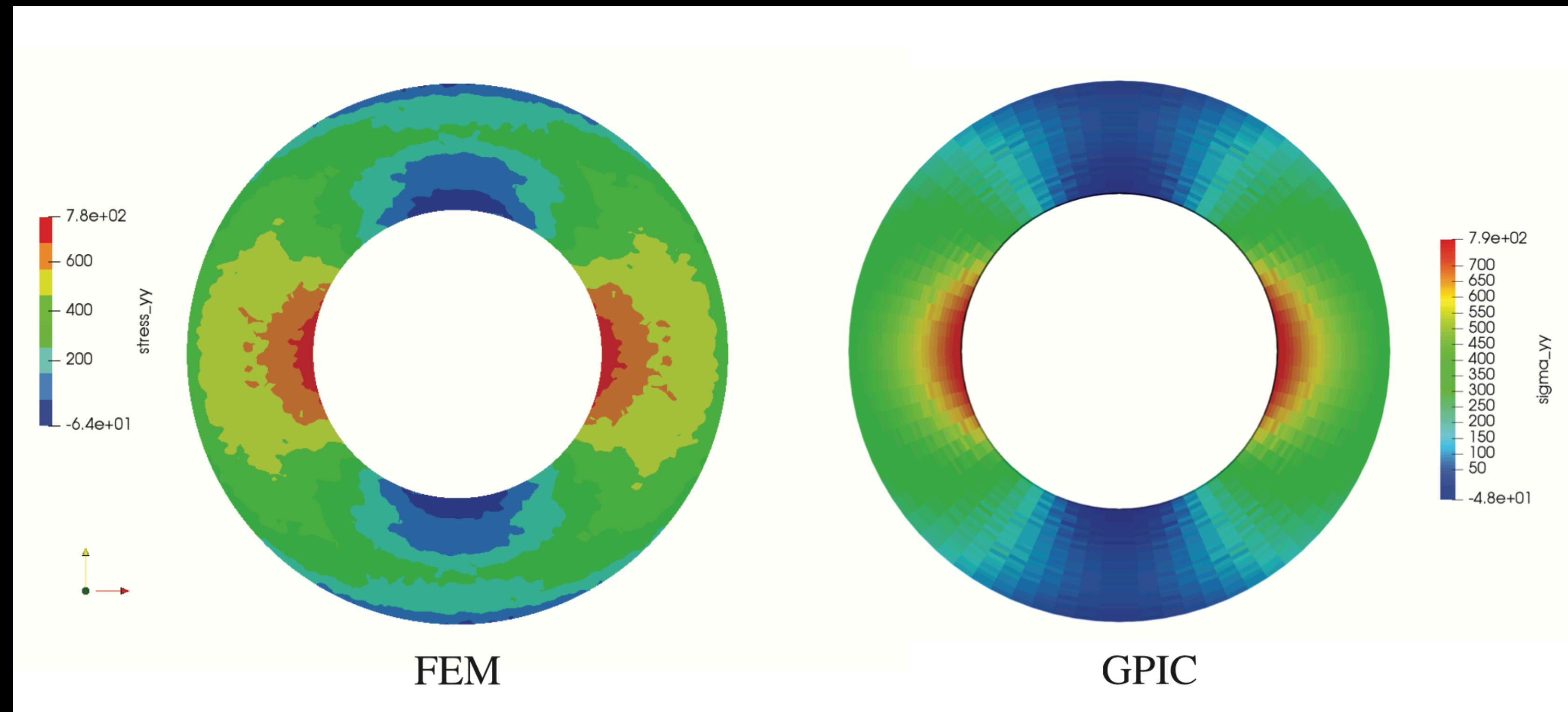
GPIC

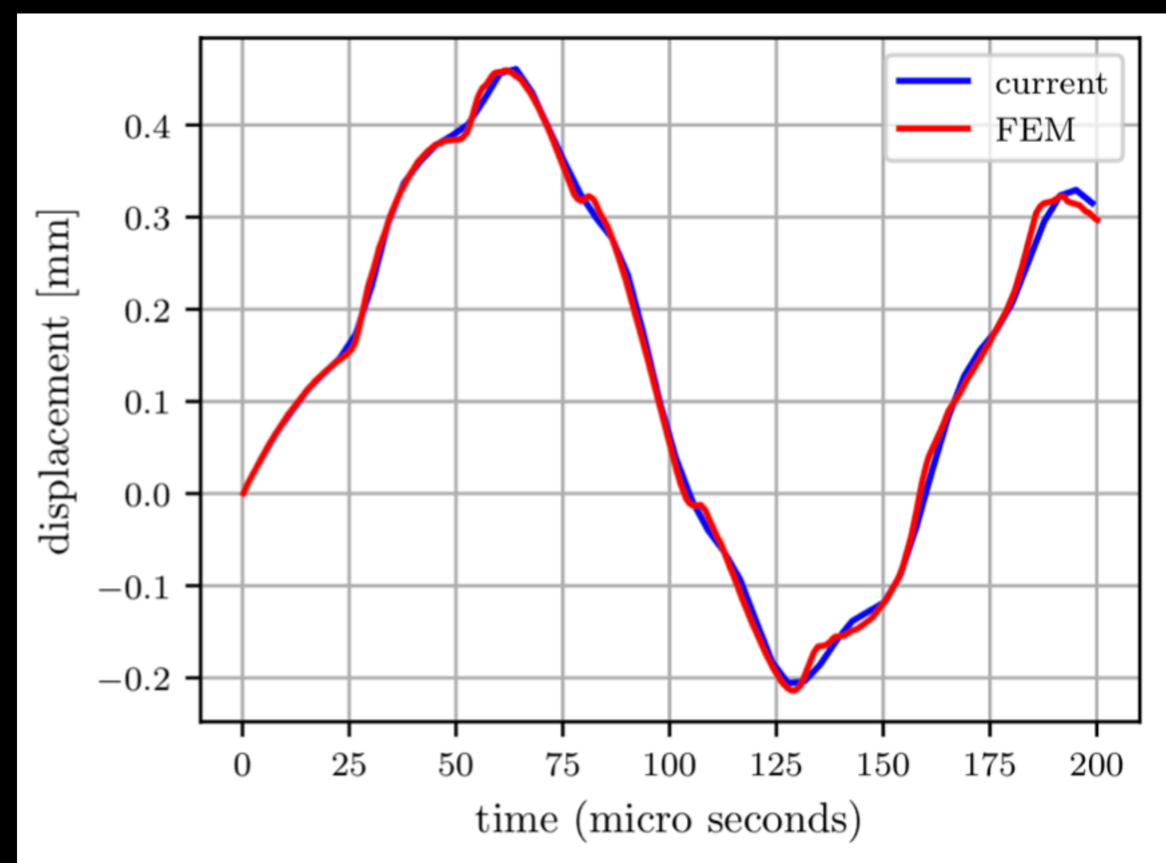


Generalised Particle in Cell (GPIC): example 2

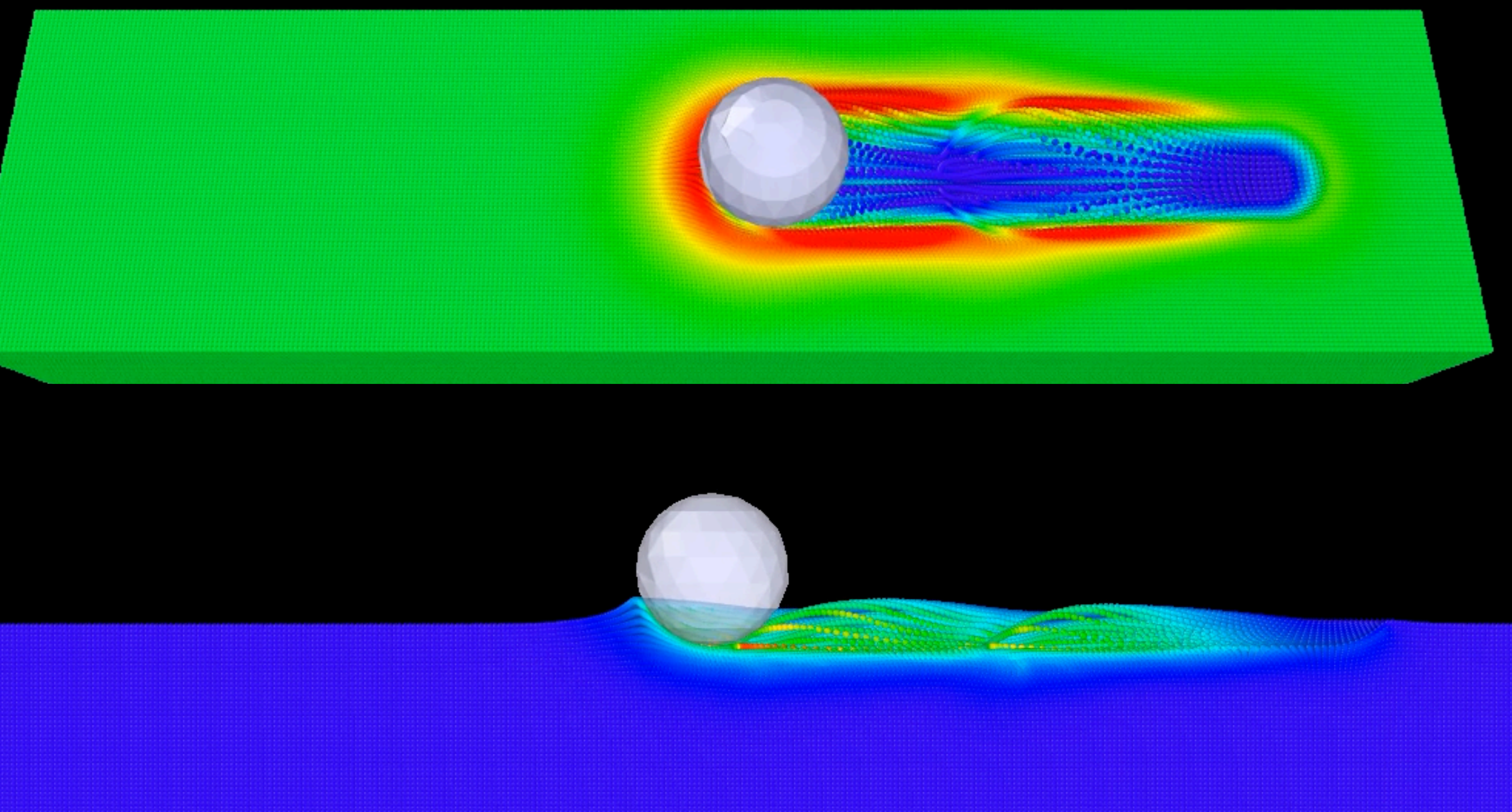




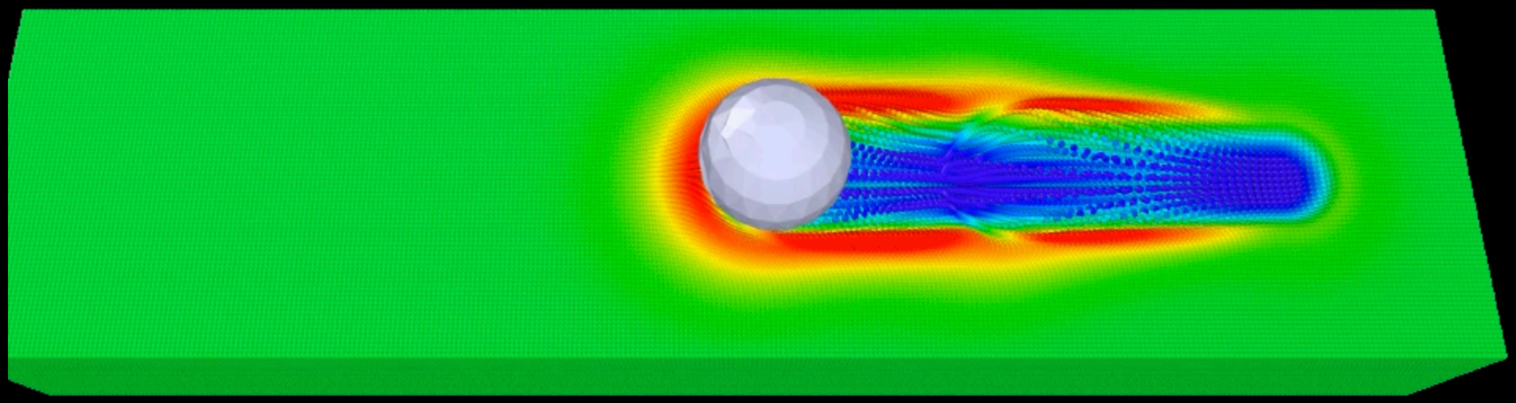


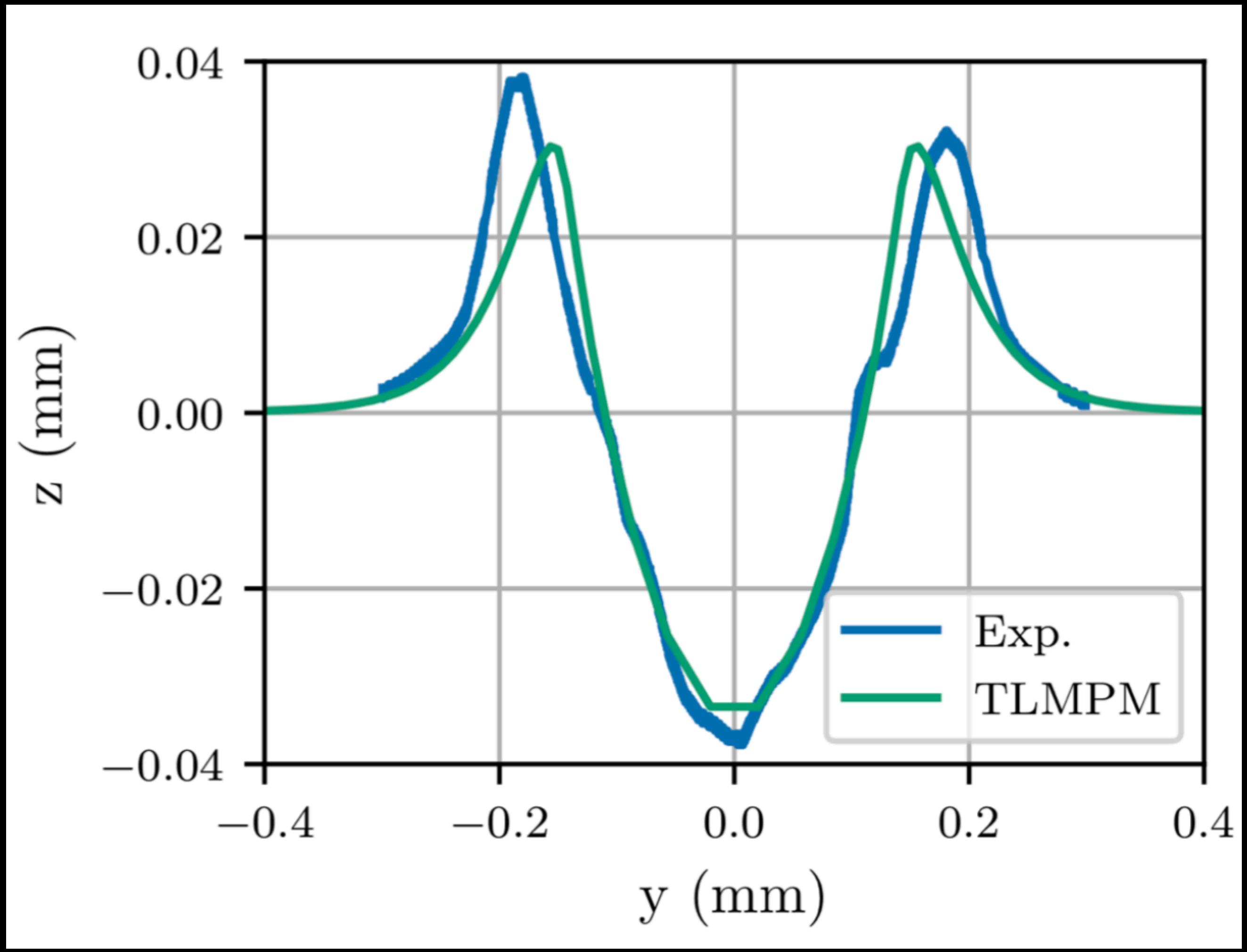


Scratch test of copper: TLMPM, Johnson-Cook plastic model



Scratch test of copper: groove topology





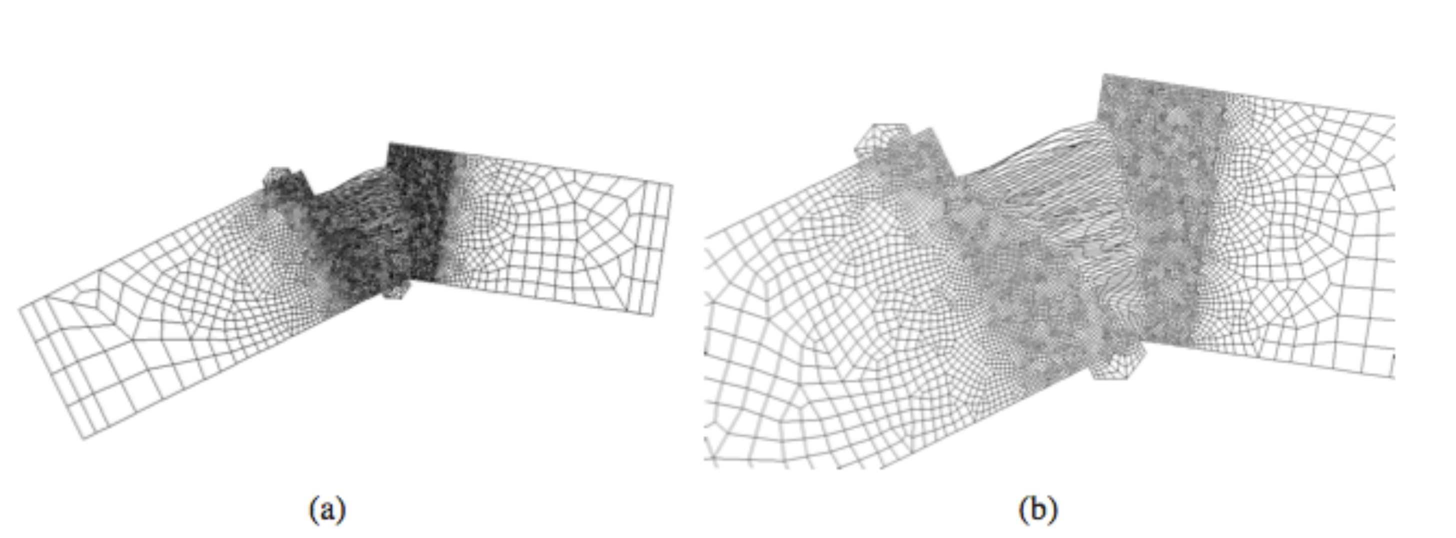
Modelling fracture

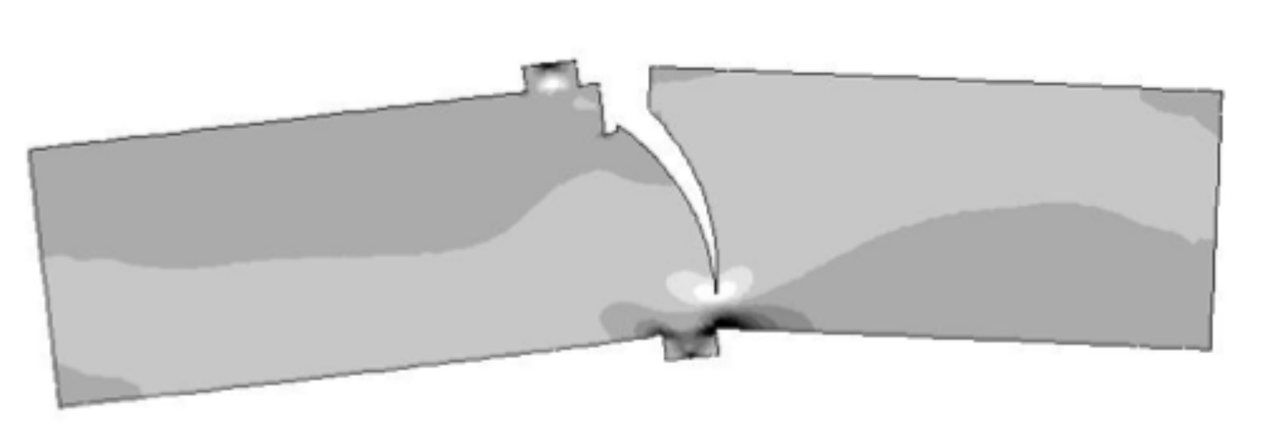
Damage mechanics approach:

+ easy to code (2D, 3D)
- less efficient (refined meshes)
- hard to capture discontinuities
- required non locality

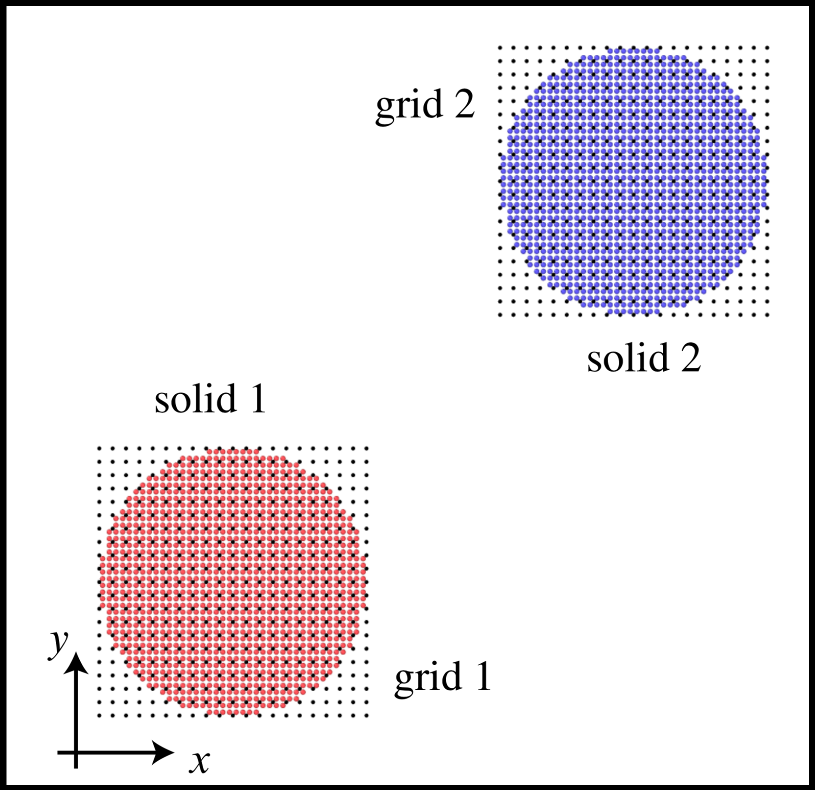
Fracture mechanics approach:

+ capture discontinuities
+ efficient
- hard to implement
- tracking 3D non-planar crack surfaces





TLMPM: modeling ductile fracture

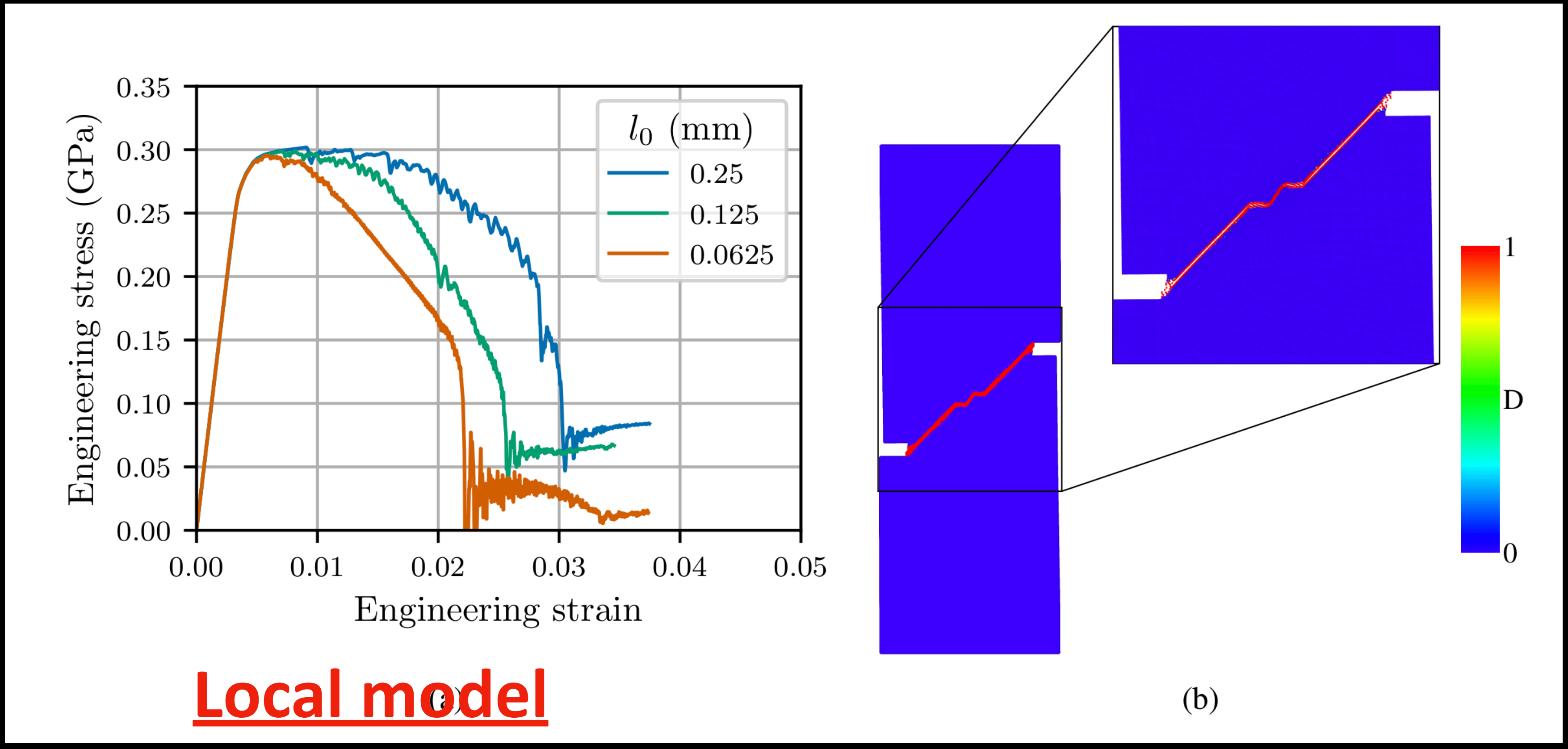


GRID

Damage mechanics approach

Local models

Nonlocal models



Local model

mesh dependent

mesh biased

Nonlocal gradient enhanced JC damage: length scale

$$\Delta D_{\text{init}} = \frac{\Delta \varepsilon_p}{\varepsilon_f} \qquad \varepsilon_f = \left(D_1 + D_2 \exp(D_3 \sigma^*)\right)\left(1 + \dot{\varepsilon}_p^*\right)^{D_4}$$

$$\boxed{\langle \Delta D^{\text{init}} \rangle - c_0 \nabla_0^2 \langle \Delta D^{\text{init}} \rangle = \Delta D^{\text{init}}} \qquad c_0 \sim l_d^2$$

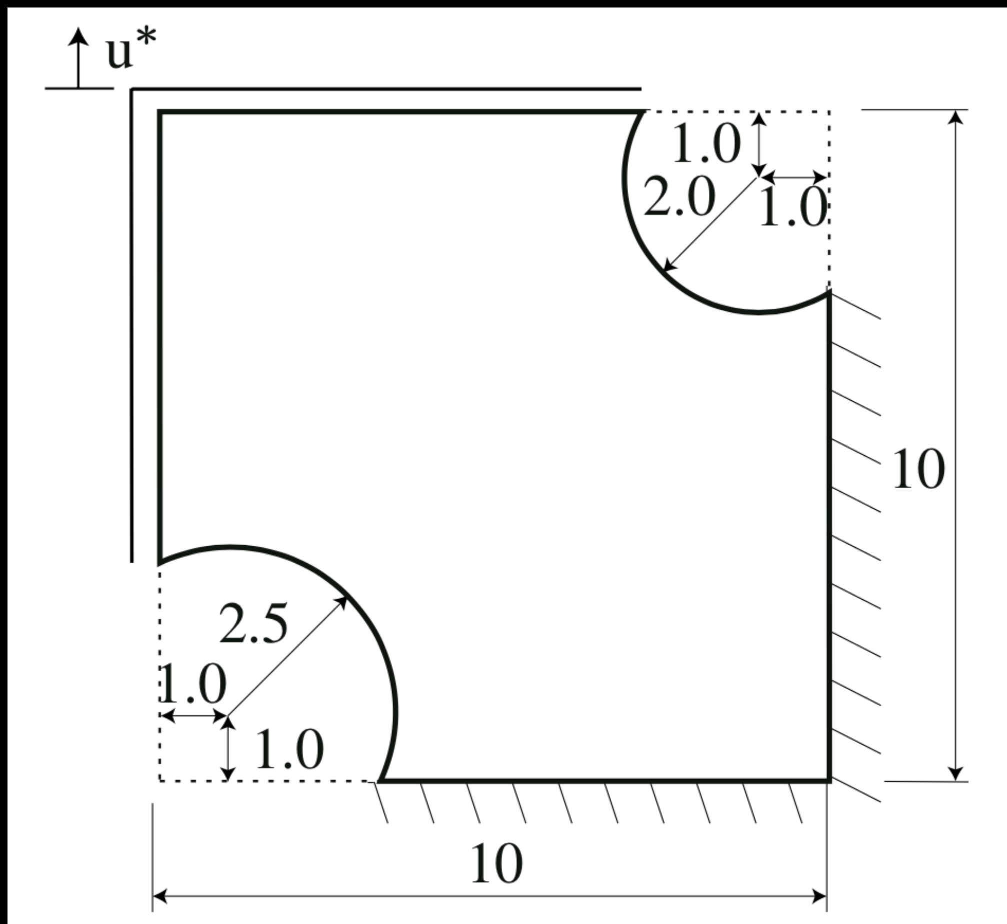
$$\langle D_{\text{init}} \rangle := \sum \langle \Delta D_{\text{init}} \rangle$$

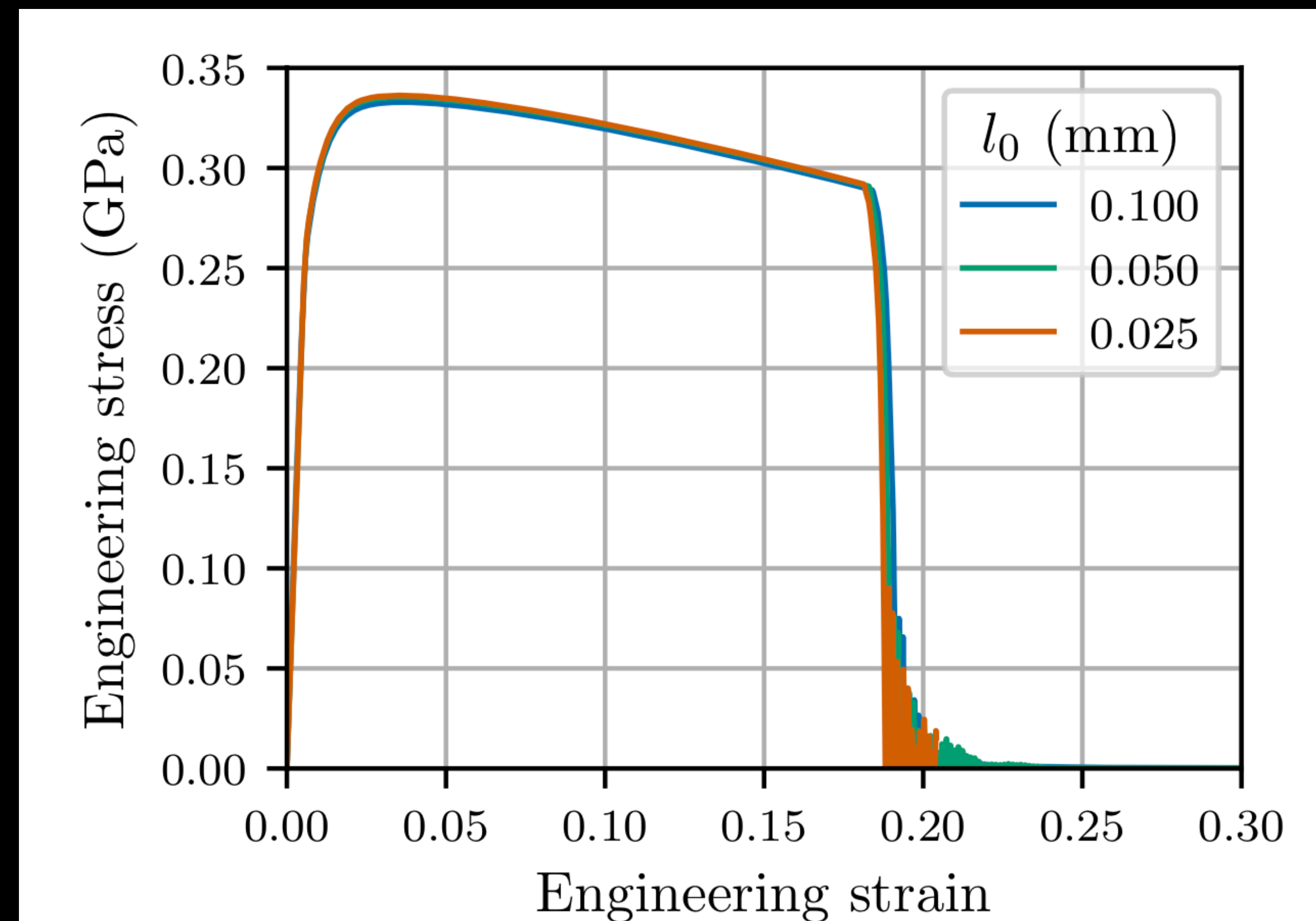
$$D = \begin{cases} 0 & \text{when } 0 \le \langle D_{\text{init}} \rangle < 1 \\ 10\left(\langle D_{\text{init}} \rangle - 1\right) & \text{when } \langle D_{\text{init}} \rangle \ge 1 \end{cases}$$

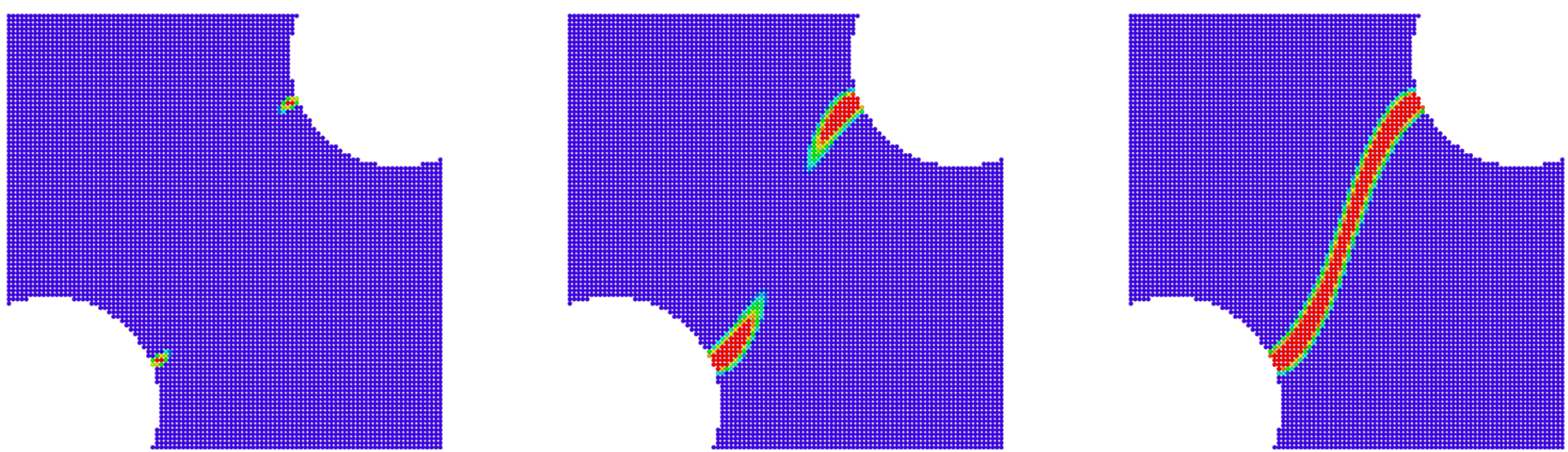
$$\boldsymbol{\sigma} = (1 - D)\bar{\boldsymbol{\sigma}}$$

R. H. J. Peerlings, R. de Borst, W. A. M. Brekelmans, and J. de Wree. Gradient enhanced damage for quasi brittle materials. International Journal for Numerical Methods in Engineering, 39:3391–3403, 1996

TLMPM—modelling ductile fracture: result 1

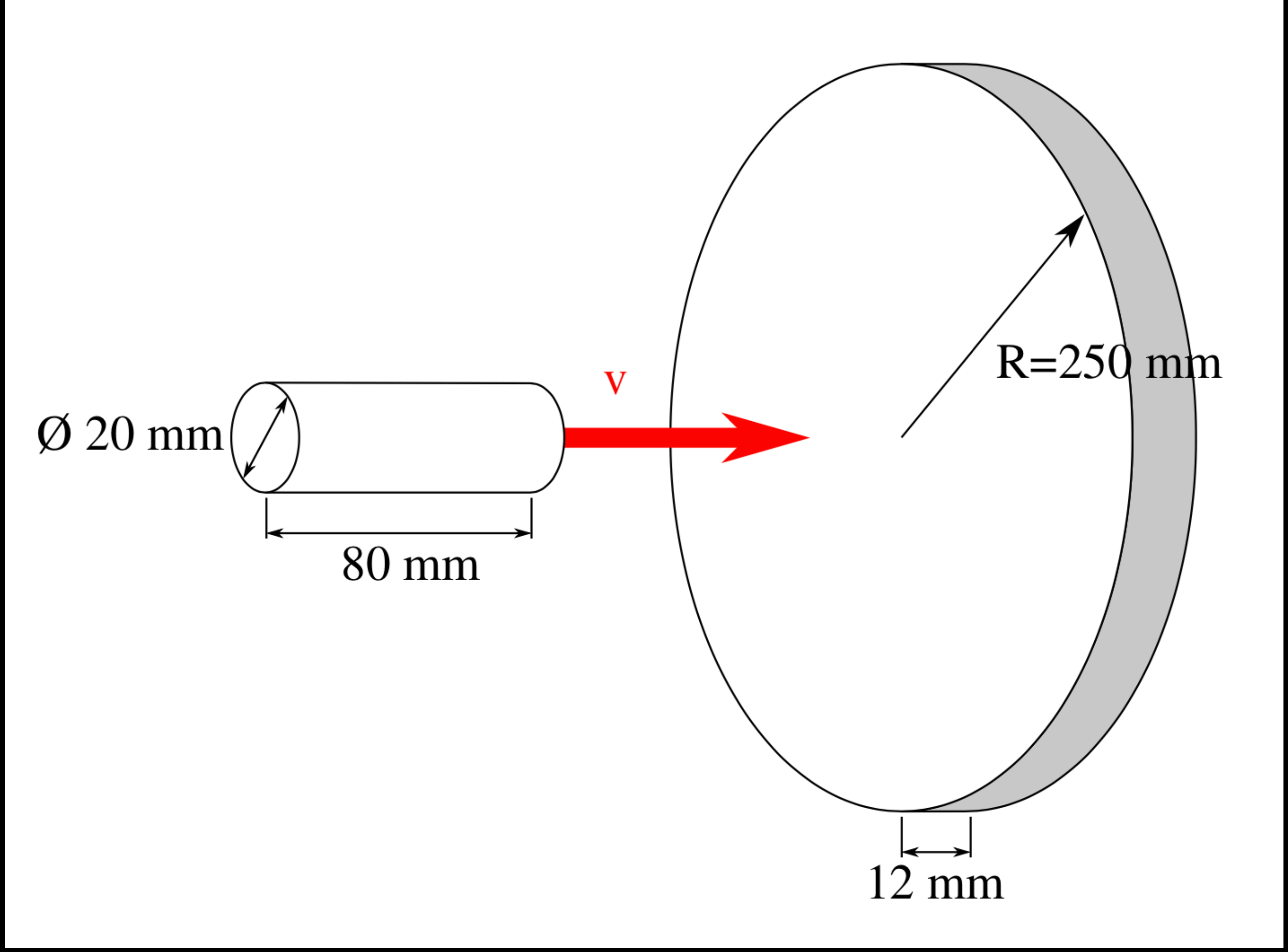




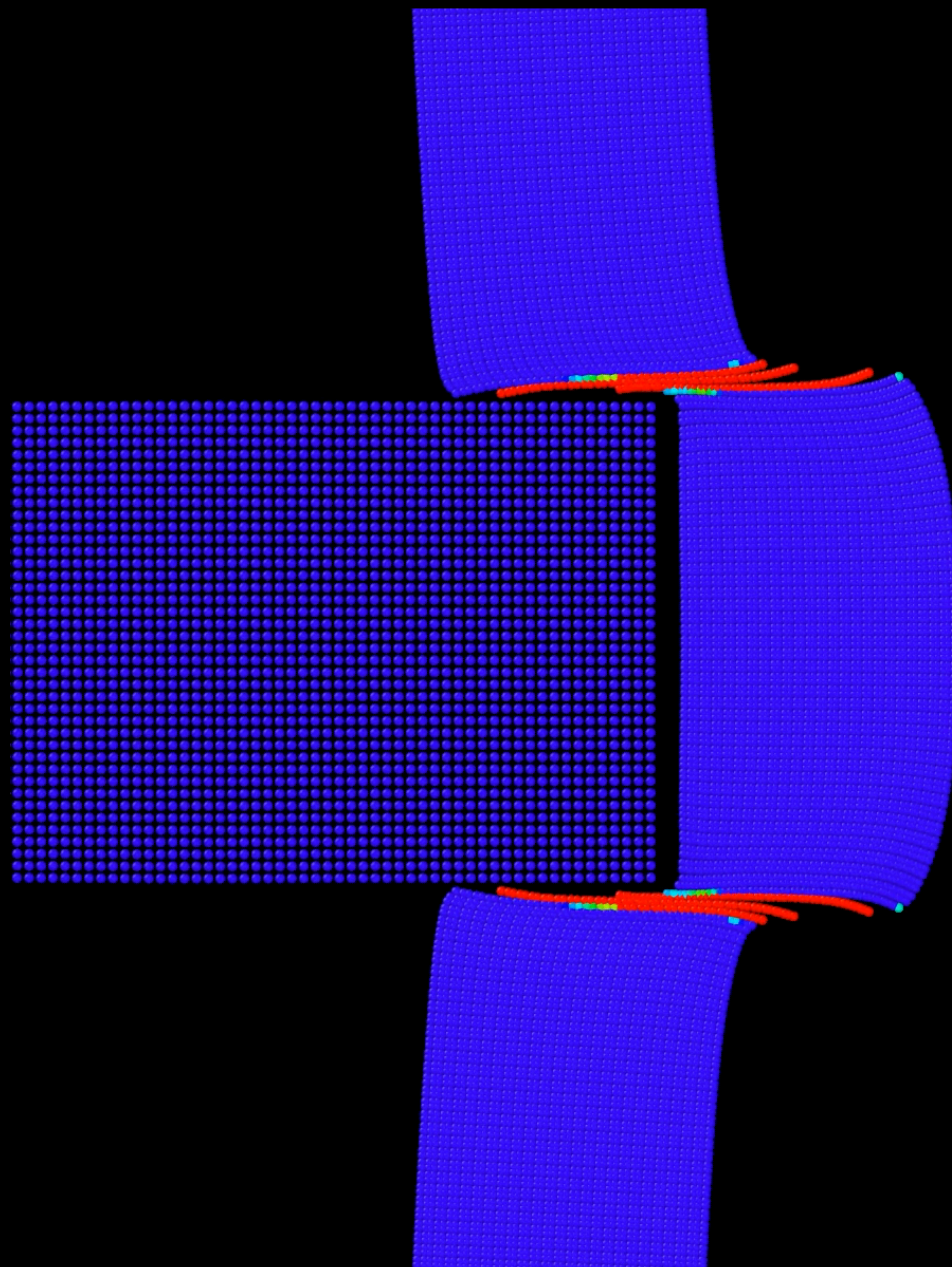


(a) $\varepsilon_p = 1.069$ (b) $\varepsilon_p = 1.583$ (c) $\varepsilon_p = 1.584$

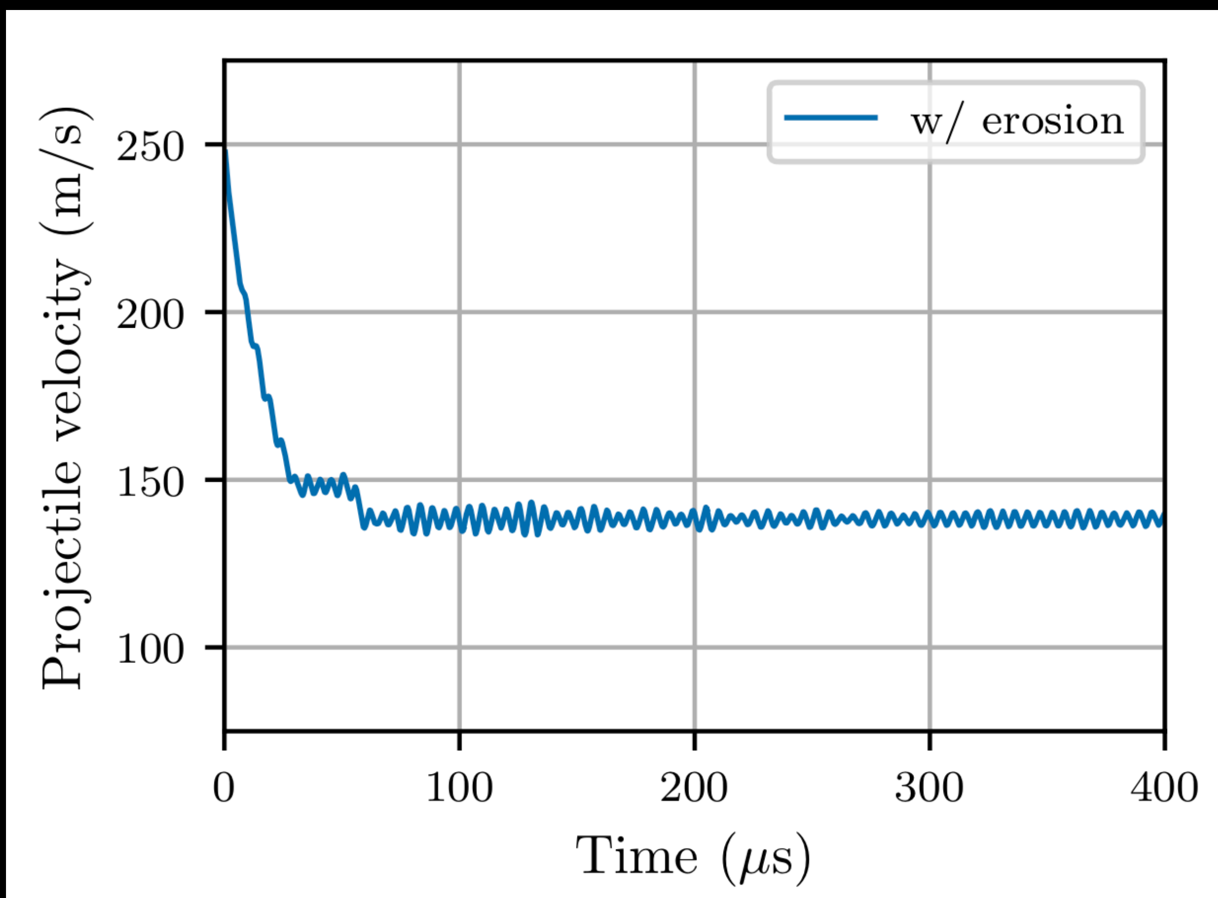
TLMPM—modelling ductile fracture: result 2

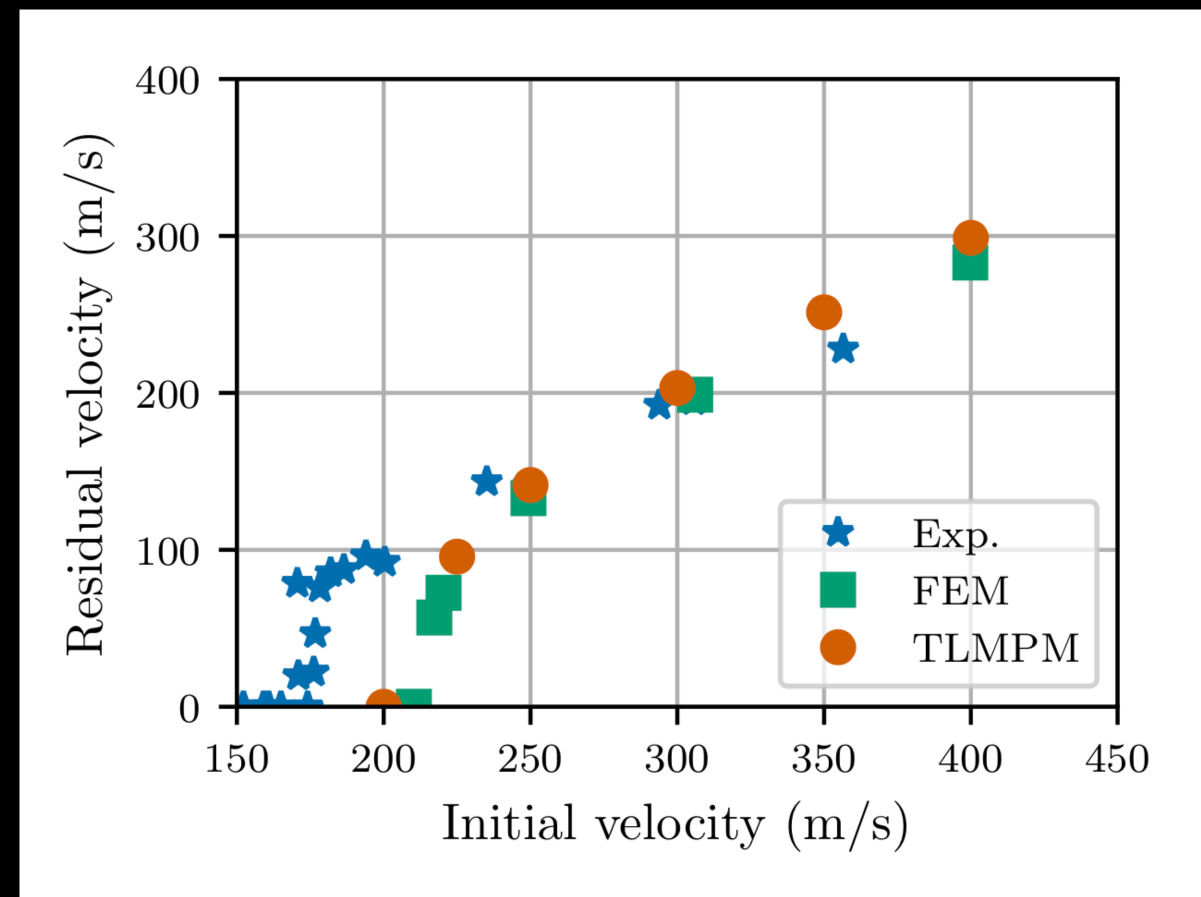


TLMPM—modelling ductile fracture: result 2

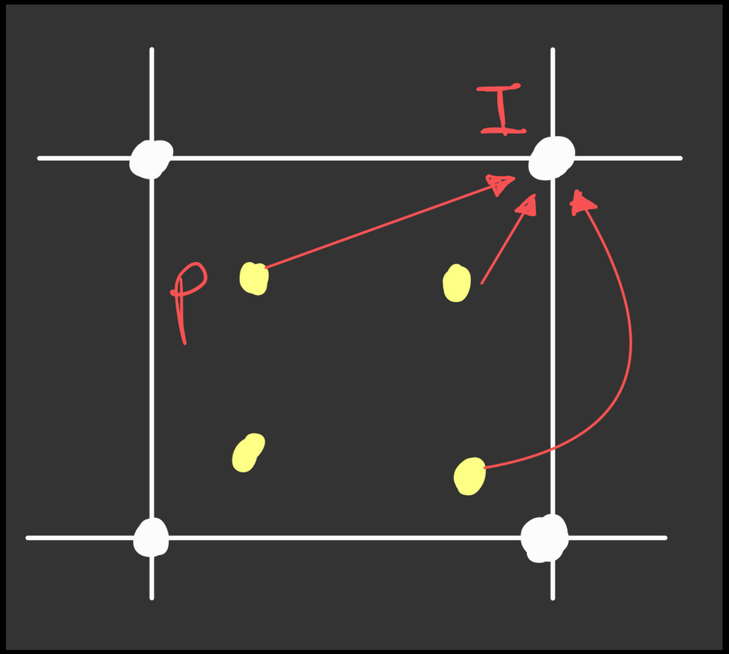


TLMPM—modelling ductile fracture: result 2





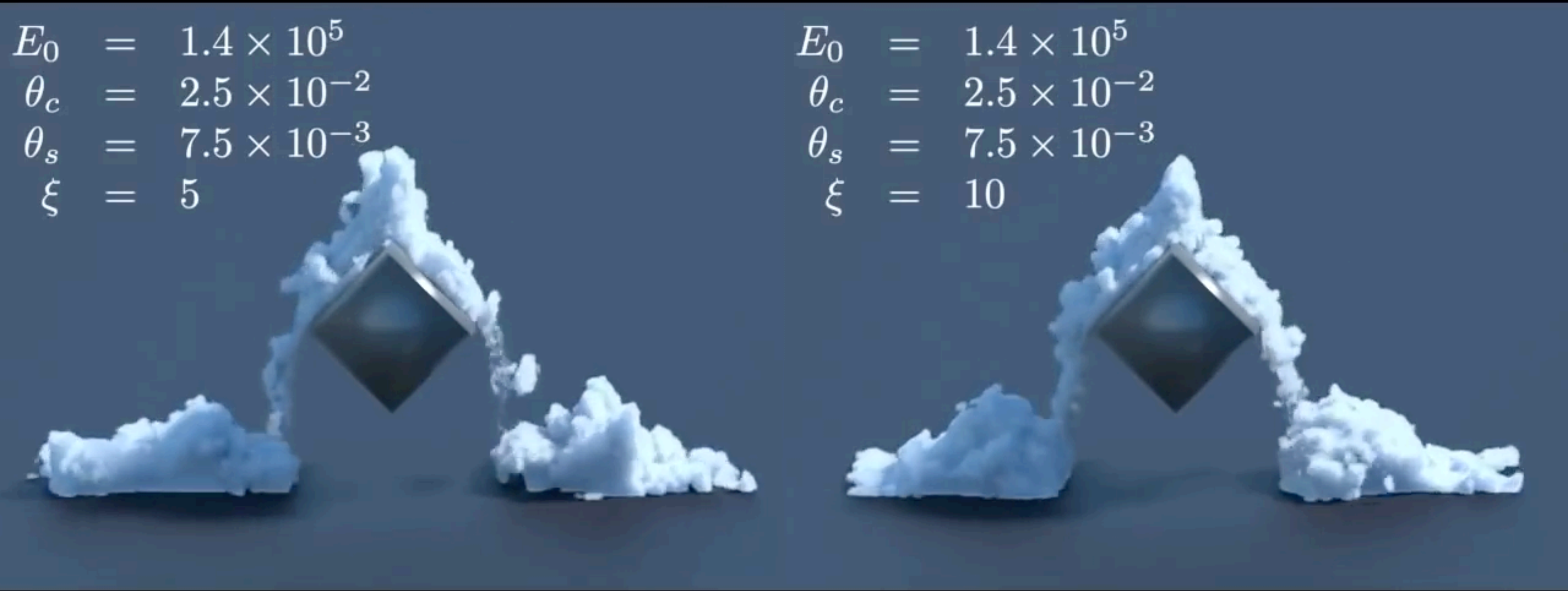
Summary



no mesh generation
simple basis functions
large deformation
multiple contacts
simple BC treatment
simple implementation

approximate geometry
blur material interfaces
numerical fracture: ULMPM

Effect of hardening



The end

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