An introduction to isogeometric analysis

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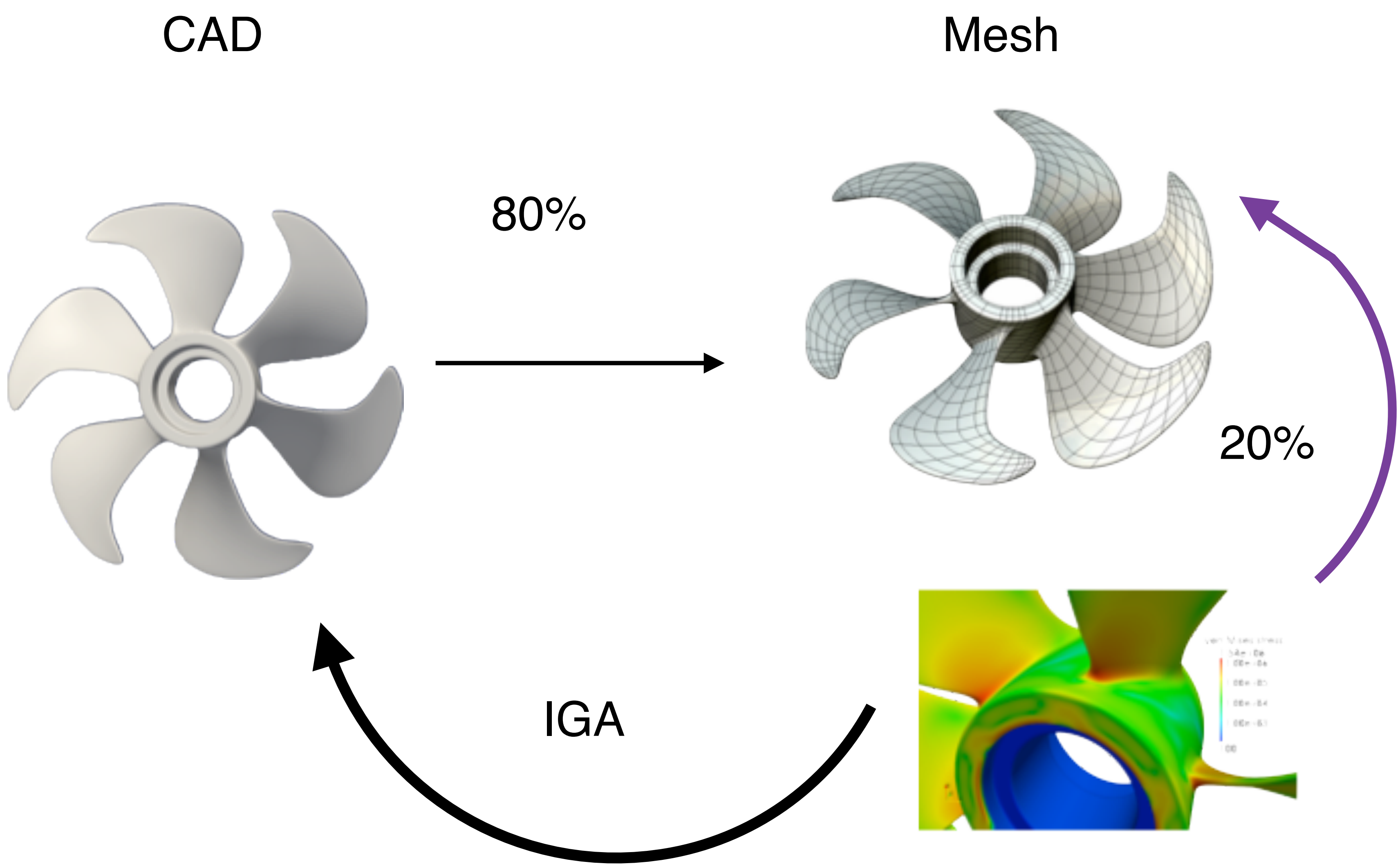
How Isogeometric analysis was born?



MA Scott et al, CMAME 2013.

B-splines basis functions

$$\Xi = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}$$ knot vector

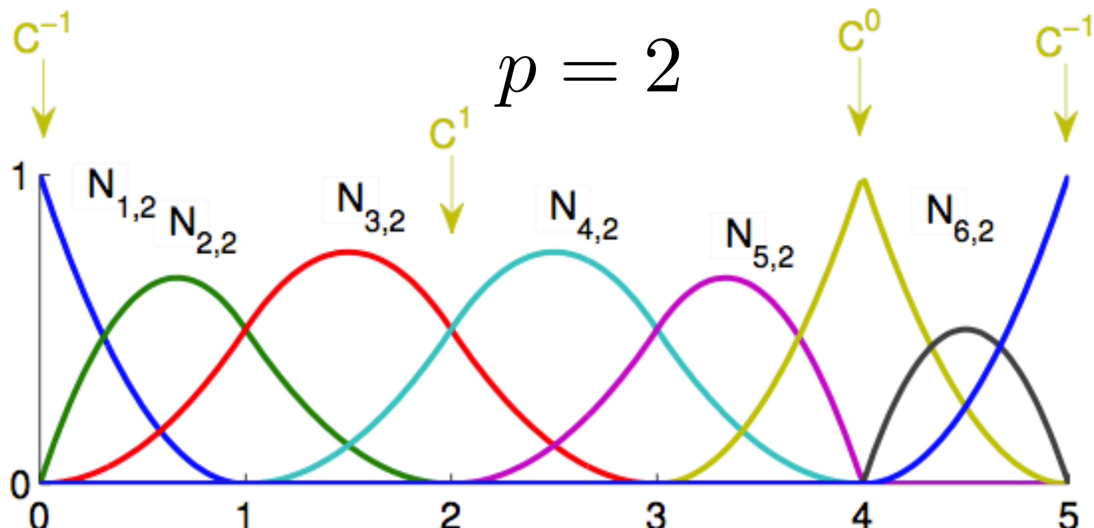
$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \leq \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

\sigma

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$

Properties

- Partition of Unity
- Linear independence
- Non-negativity
- C^{p-m} continuity
- Not interpolants



$$\Xi = \{0, 0, 0, 1, 2, 3, 4, 4, 5, 5, 5\}$$

B-splines curves/surfaces

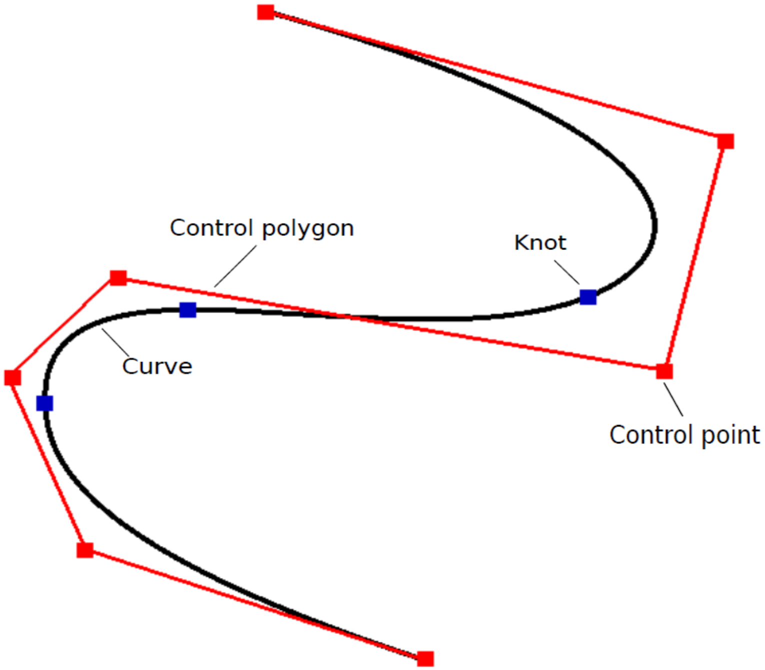
$$\Xi = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}$$

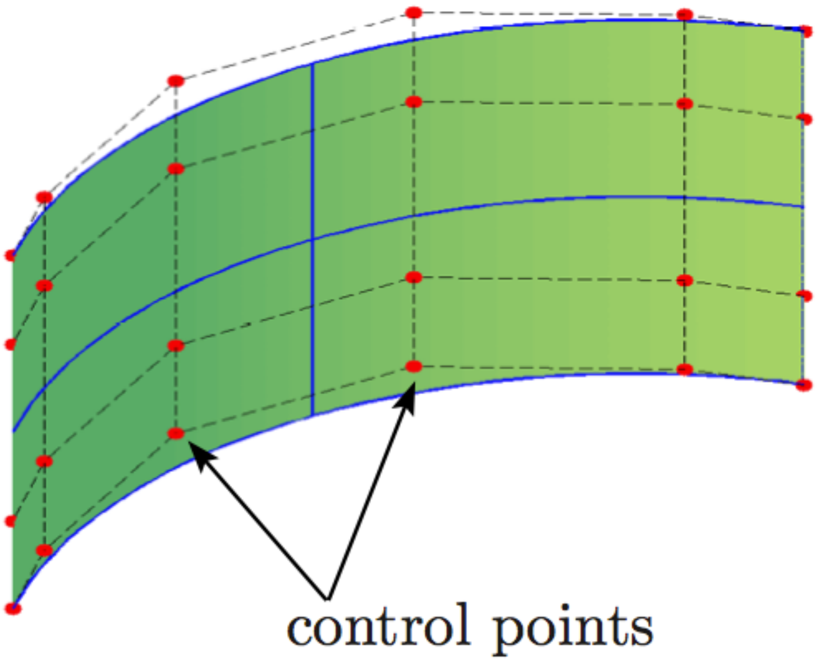
$$\Xi^1 = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}$$

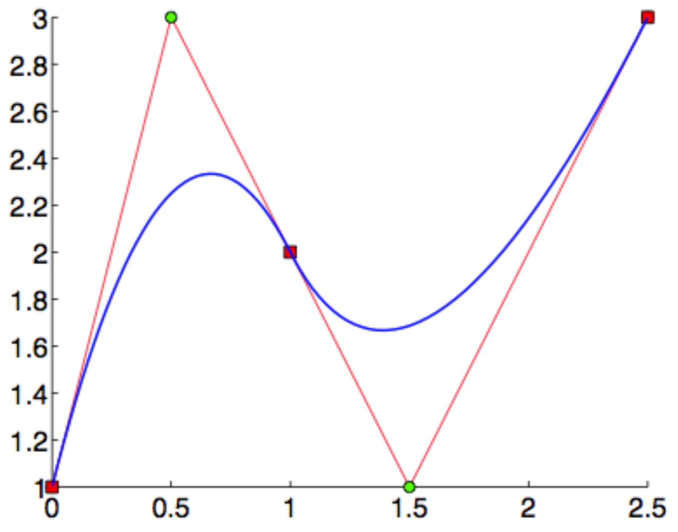
$$\Xi^2 = \{\eta_1, \eta_2, \ldots, \eta_{m+q+1}\}$$

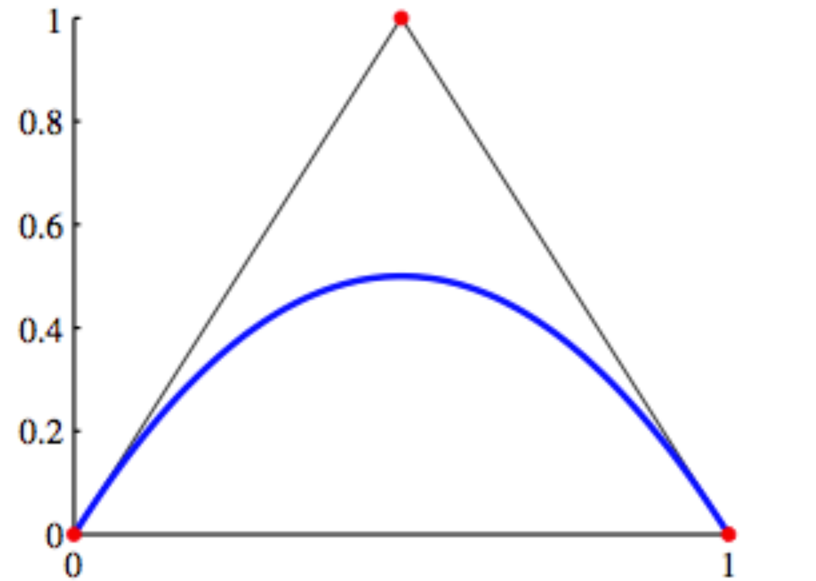
$$\mathbf{C}(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi)\mathbf{B}_i$$

$$\mathbf{S}(\xi, \eta) = \sum_{i=1}^{n}\sum_{j=1}^{m} N_{i,p}(\xi)M_{j,q}(\eta)\mathbf{B}_{ij}$$

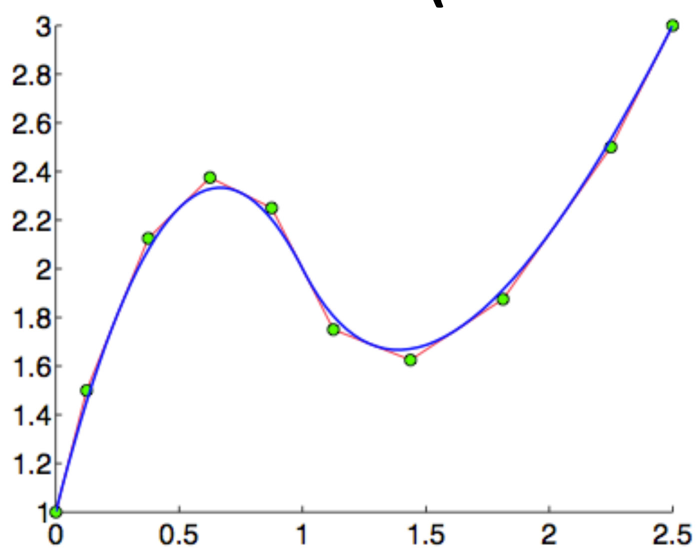


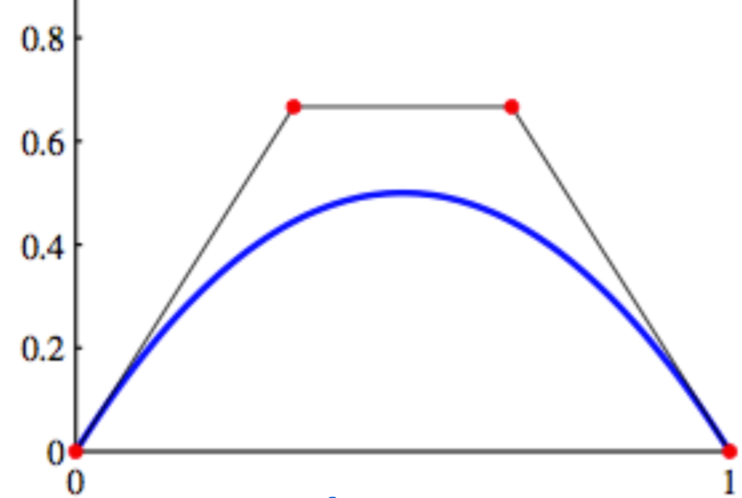






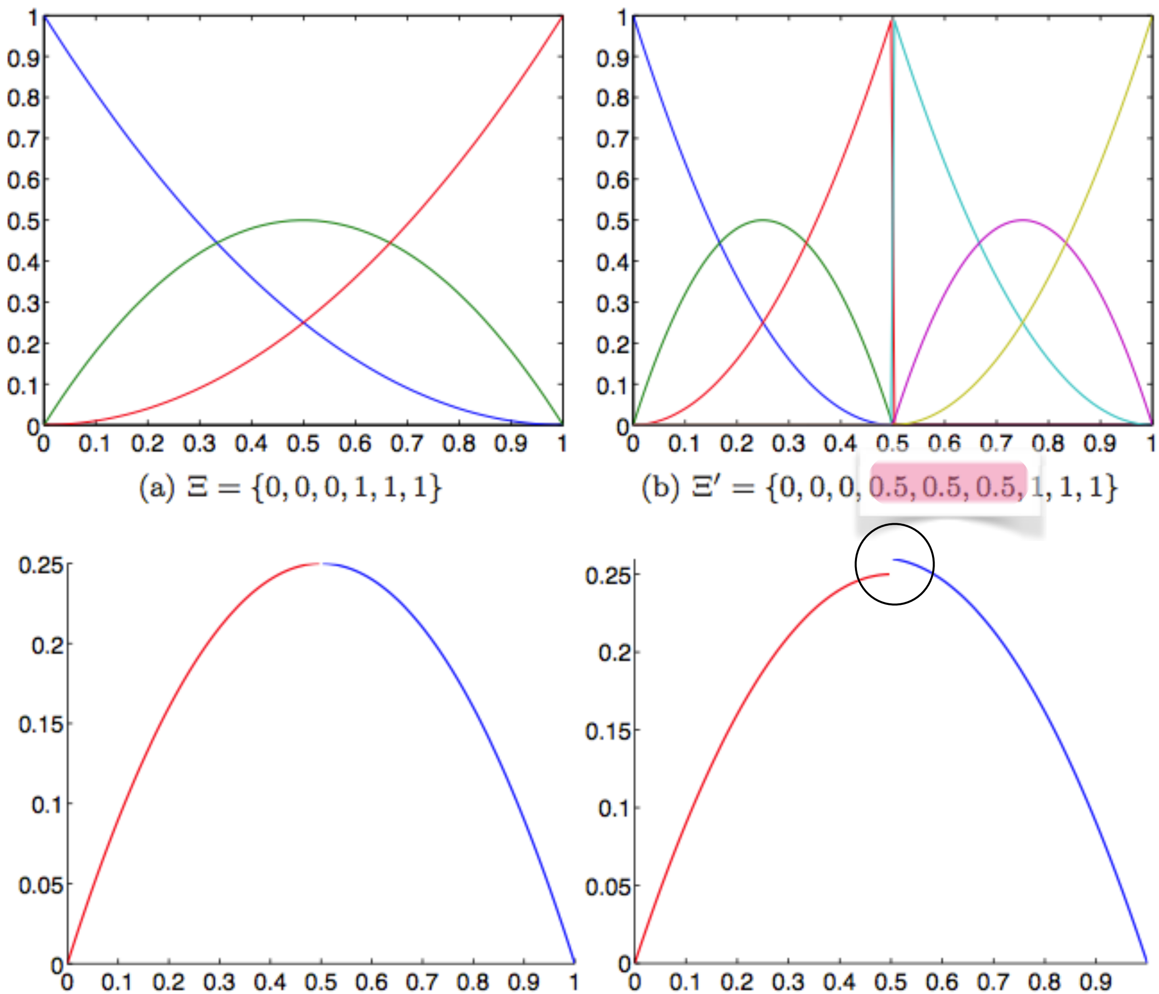
knot insertion (h-refinement) + order elevation (p-refinement)





does not change B-splines geometrically/parametrically

Knot insertion to create discontinuities

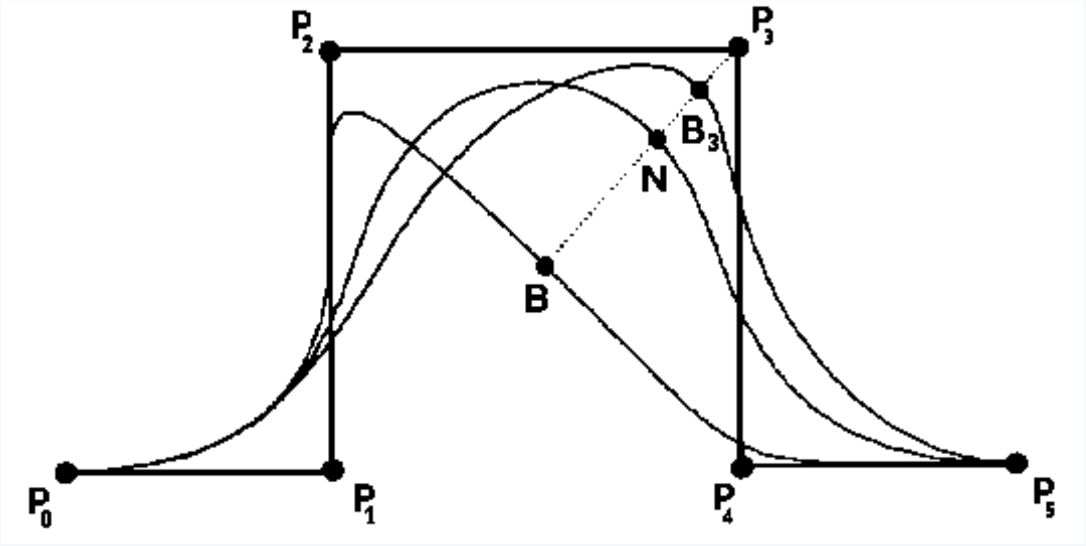


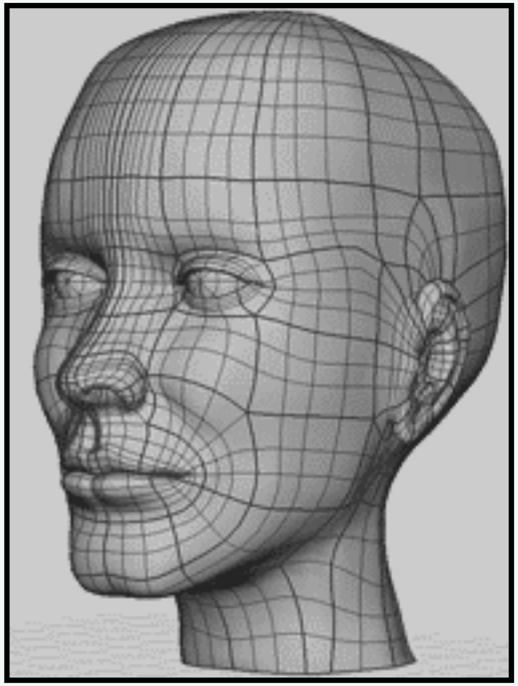
$$p = 2$$

Crack modeling and composite laminates (Layer wise theory, Z. Gurdal)

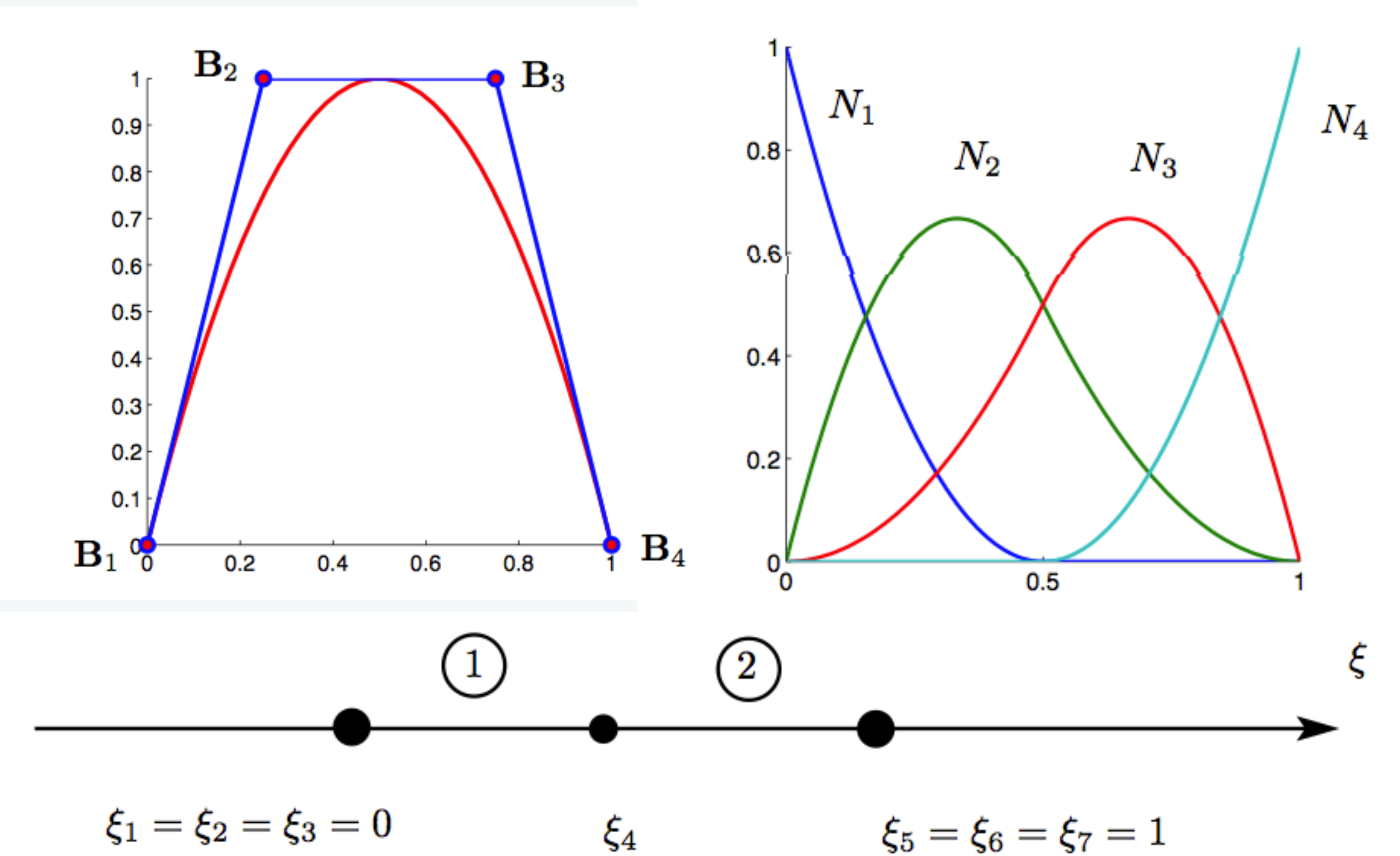
Non Uniform Rational B-splines (NURBS)

$$R_{i,p}(\xi) = \frac{N_{i,p}(\xi) w_i}{W(\xi)} = \frac{N_{i,p}(\xi) w_i}{\sum_{j=1}^{n} N_{j,p}(\xi) w_j},$$



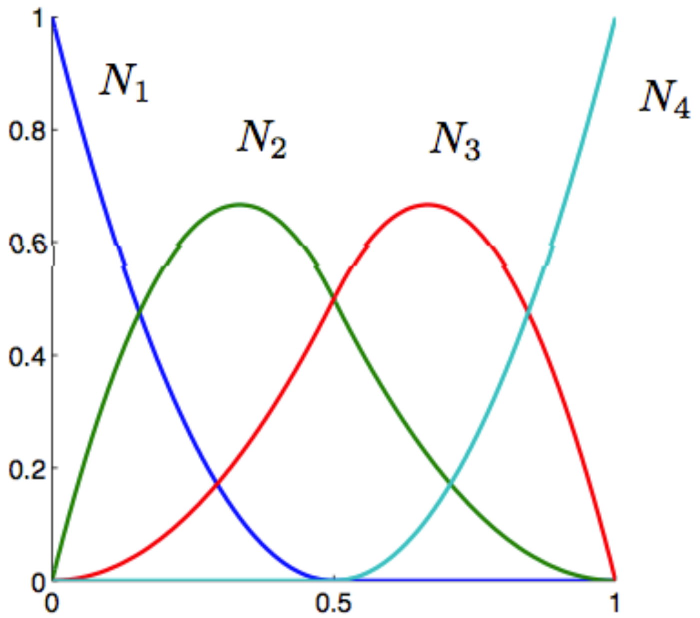


One dimensional IGA FEM



$$\Xi = \{0, 0, 0, 1/2, 1, 1, 1\}$$

One dimensional IGA FEM



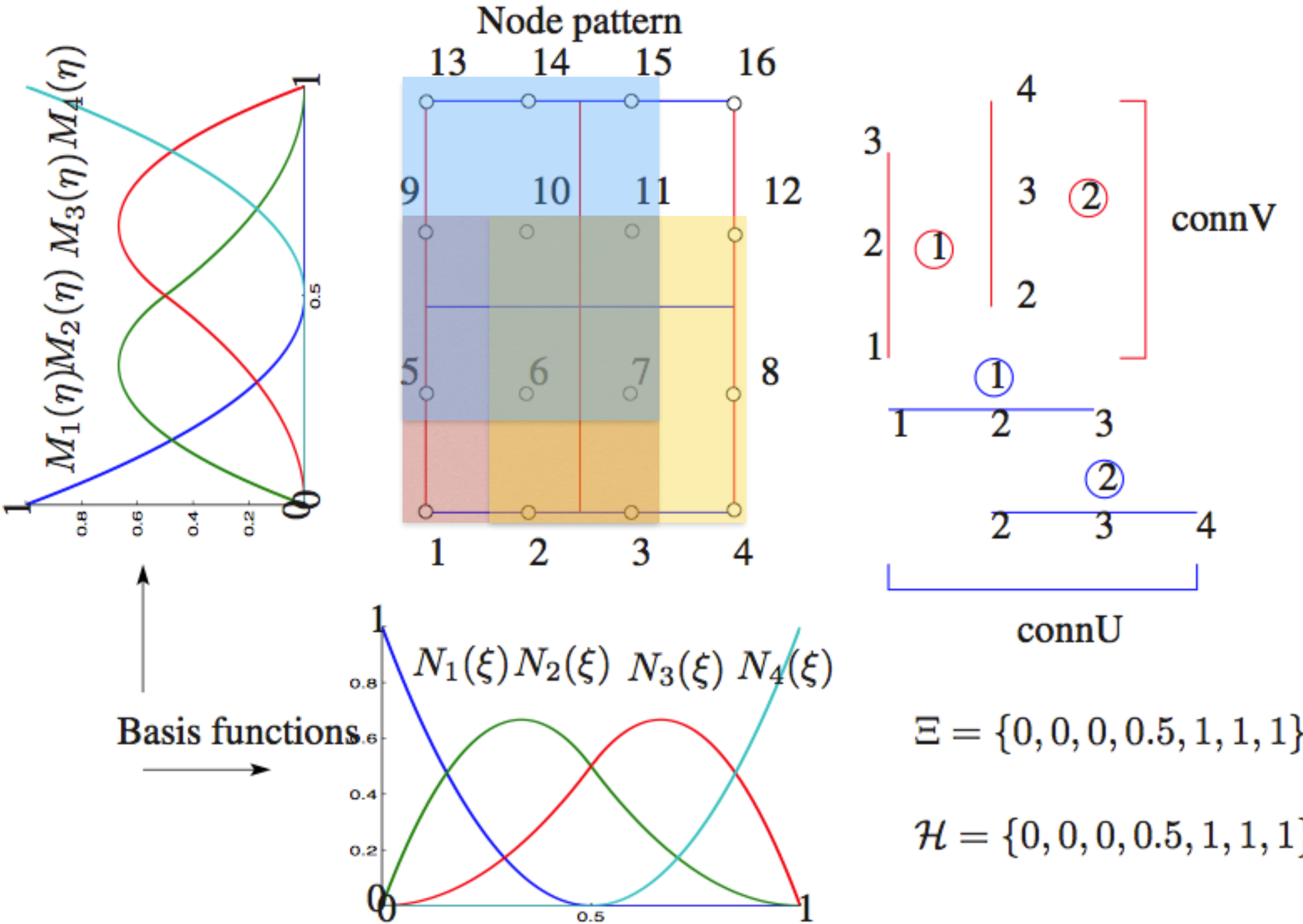
Isoparametric

$$x = N_I x_I$$

$$u = N_I u_I$$

element	range	non-zero basis	dofs	control points
1	$[\xi_3, \xi_4]$	N_1, N_2, N_3	u_1, u_2, u_3	$\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3$
2	$[\xi_4, \xi_5]$	N_2, N_3, N_4	u_2, u_3, u_4	$\mathbf{B}_2, \mathbf{B}_3, \mathbf{B}_4$

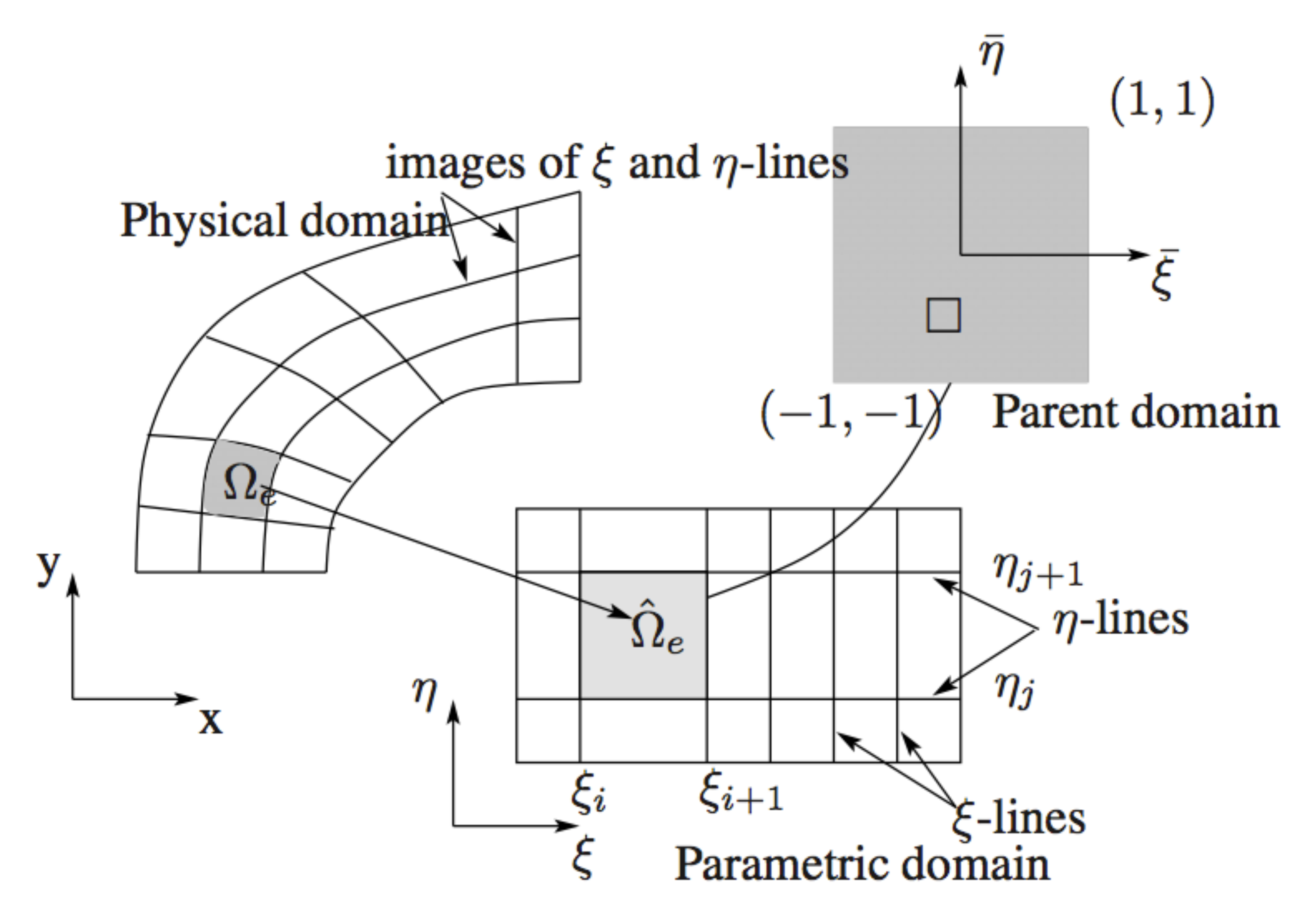
Two dimensional IGA FEM



$$\Xi = \{0, 0, 0, 0.5, 1, 1, 1\}$$

$$\mathcal{H} = \{0, 0, 0, 0.5, 1, 1, 1\}$$

Numerical integration



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Examples

Tools

MIGFEM

- quick prototyping
- tutorial codes

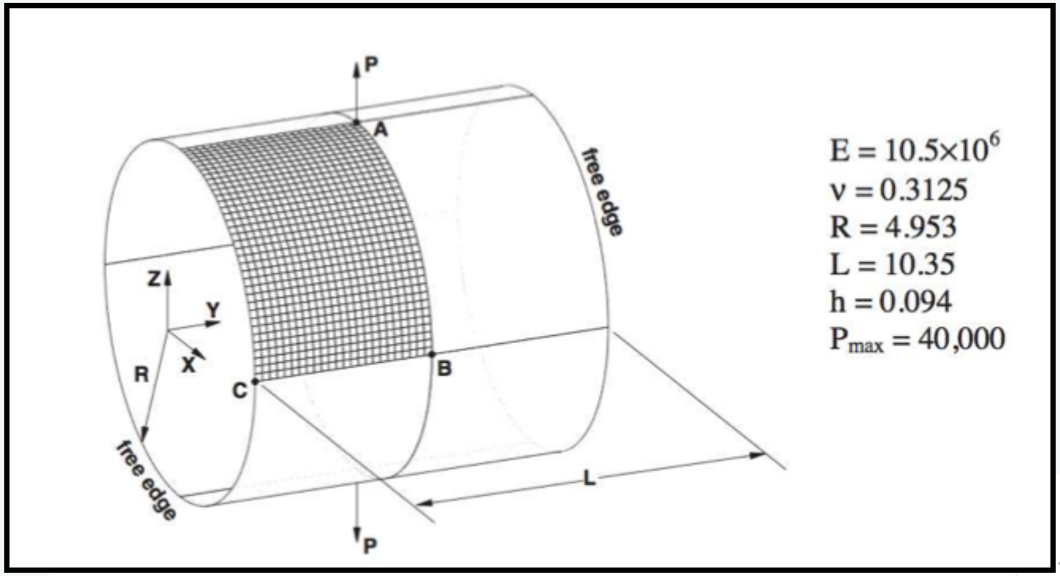
- open source Matlab Isogeometric (X)FEM
- 2D/3D solid mechanics with geometry nonlinearities
- 2D XIGA for LEFM and material interfaces
- Structural mechanics: beam, plate, shells (large deformation)
- http://sourceforge.net/projects/cmcodes/

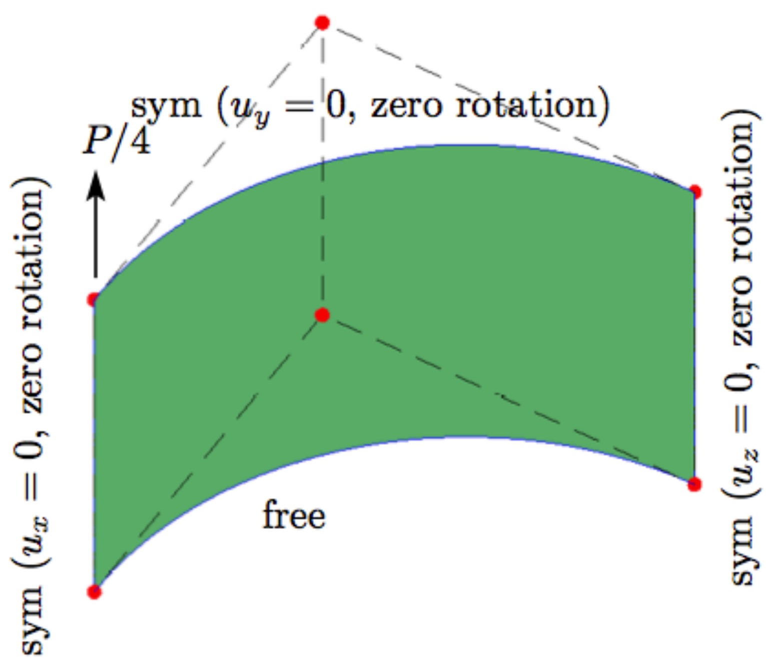
jem-jive **(Linux, Mac OS, Windows)**

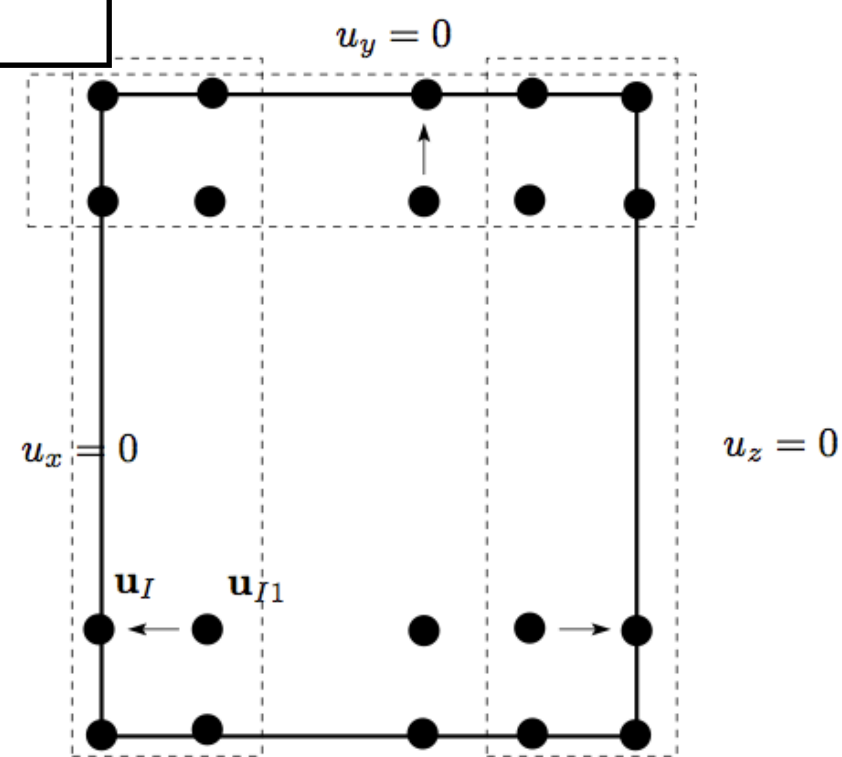
DRG Dynaflow Research Group

- commericial C++ toolkit for PDEs
- not a general purpose FE package
- tailor made applications, suitable for researchers
- apps: XFEM, dG, IGA, DEM, FVM etc.
- support parallel computing
- implements useful concepts available in other programming languages--Java, Fortran 90, Matlab and C#
- tensor class: useful to evaluating complex constitutive models
- http://www.dynaflow.com/en_GB/jive.html

IGA rotation free nonlinear shell







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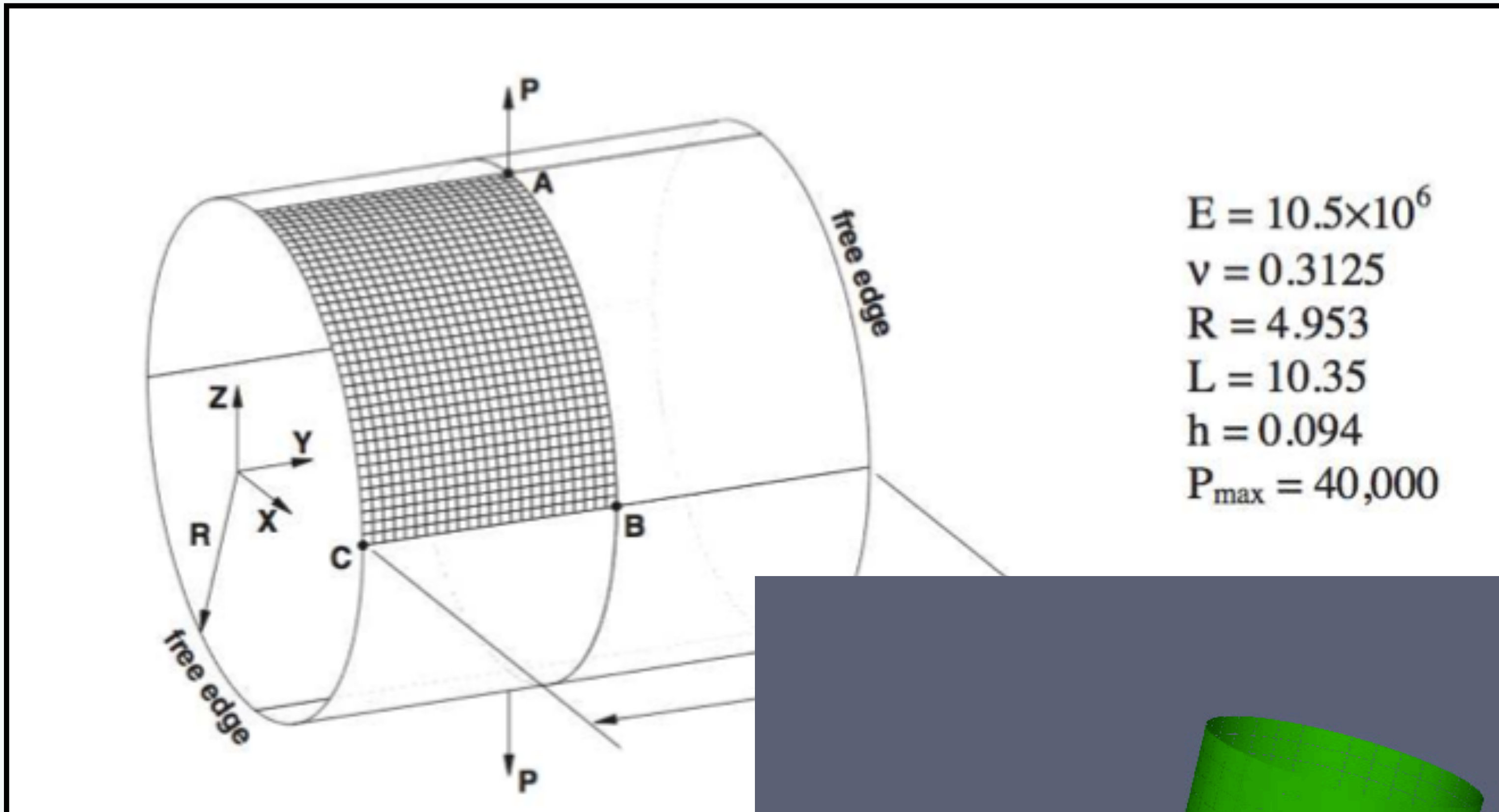
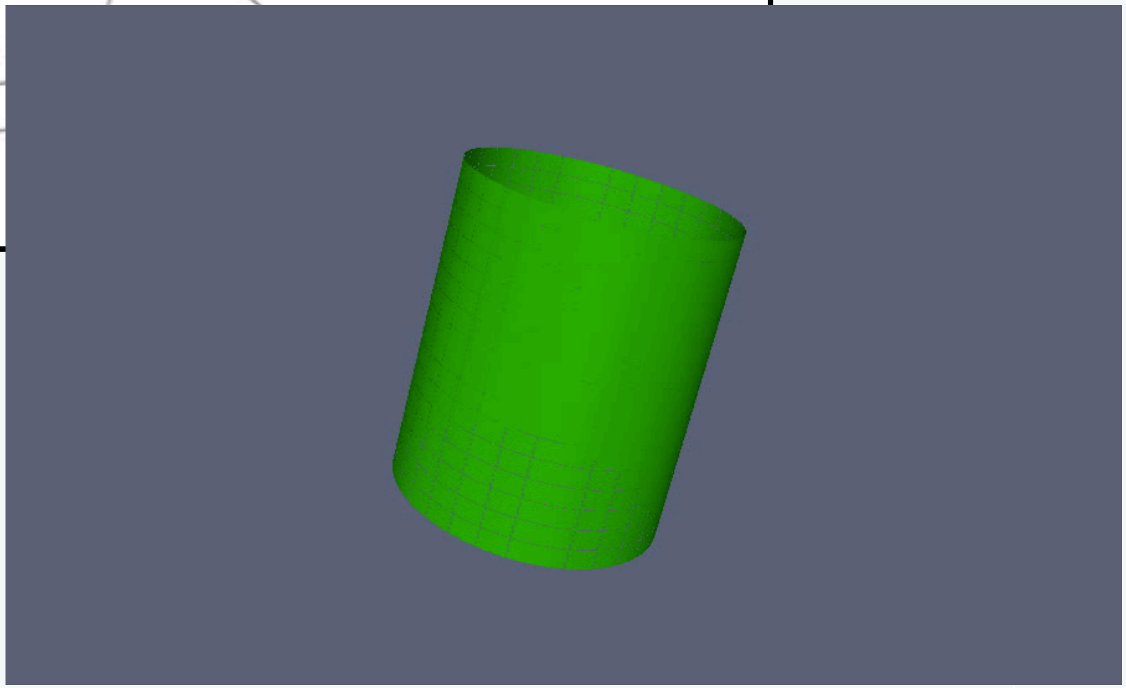
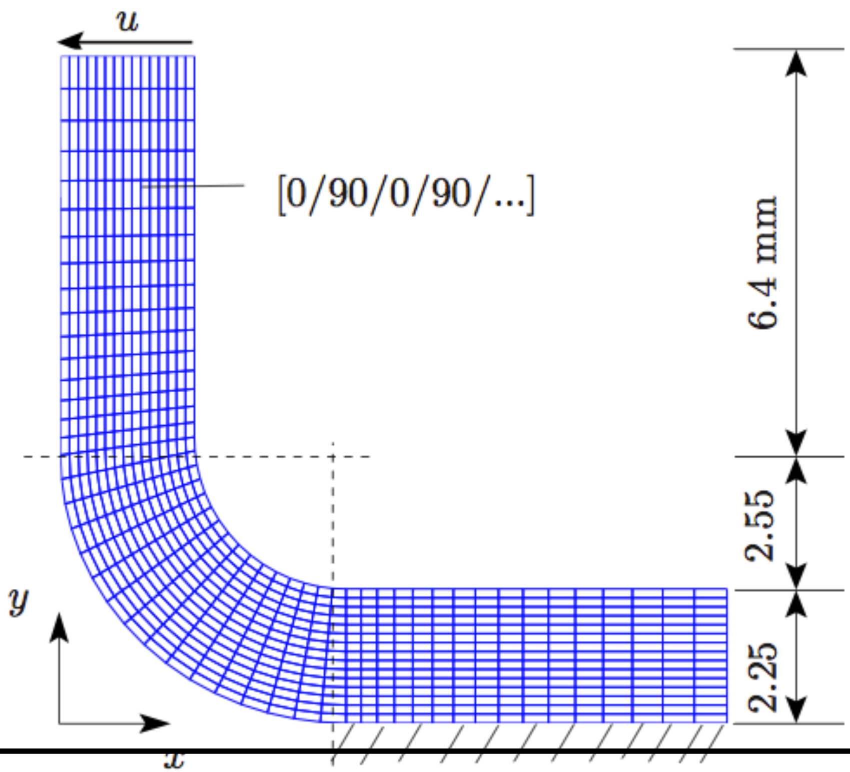
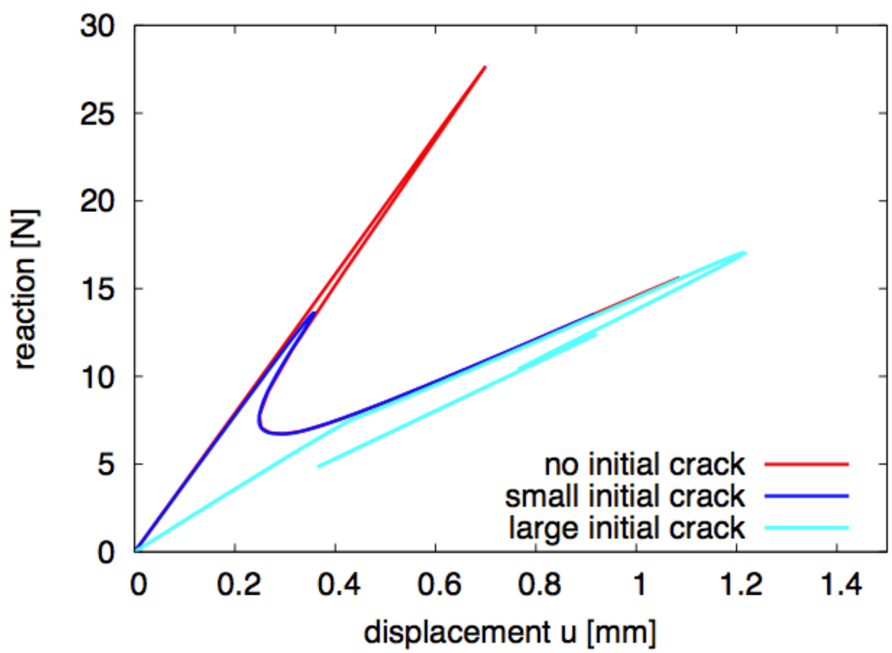




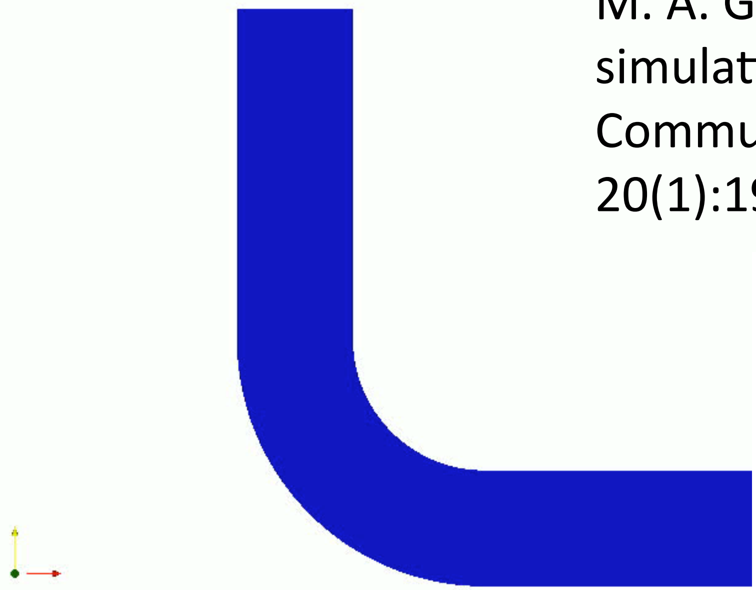
Isogeometric cohesive elements: 2D example



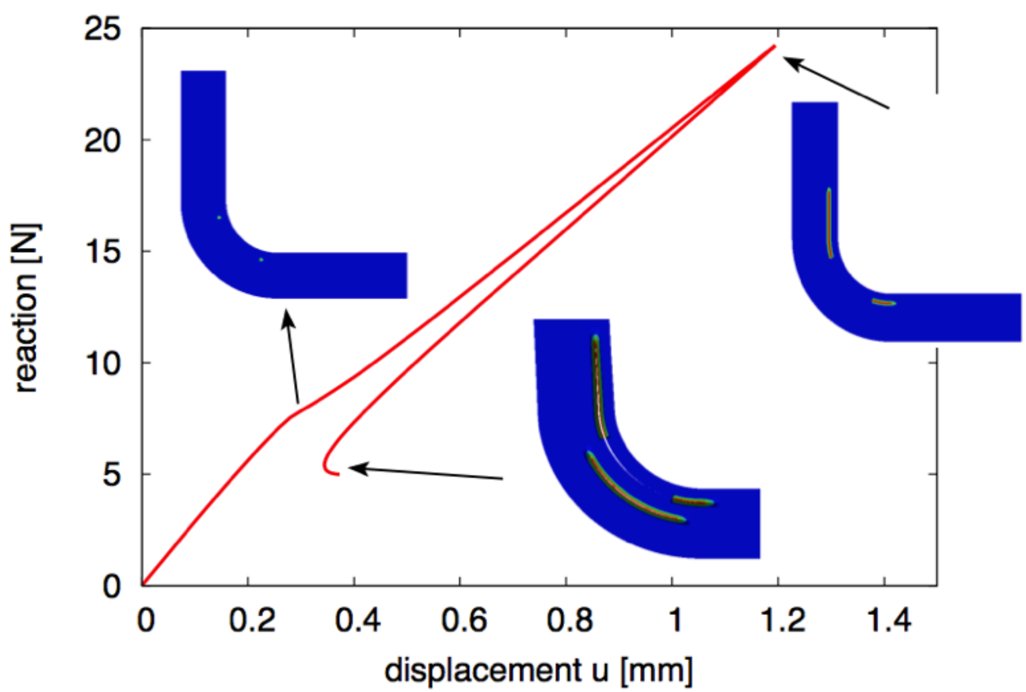


- exact geometry by NURBS
- It is straightforward to vary
  (1) number of plies and
  (2) # of interface elements:
- Suitable for parameter studies/design
- Cohesive law: bilinear law of Turon et al. 2006

Isogeometric cohesive elements: 2D example

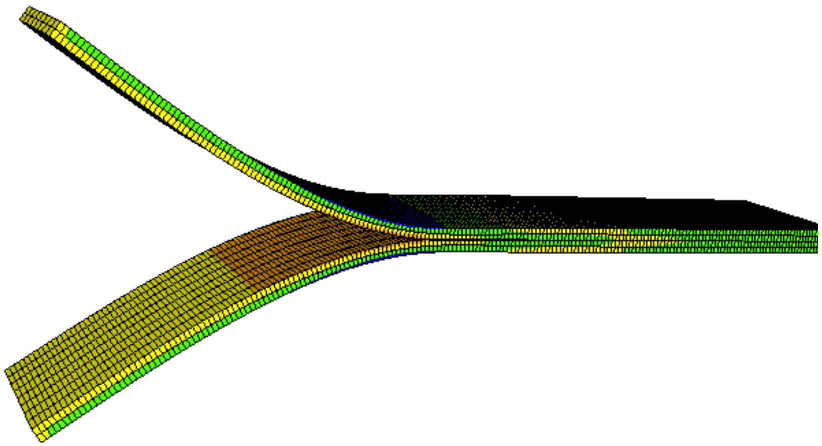


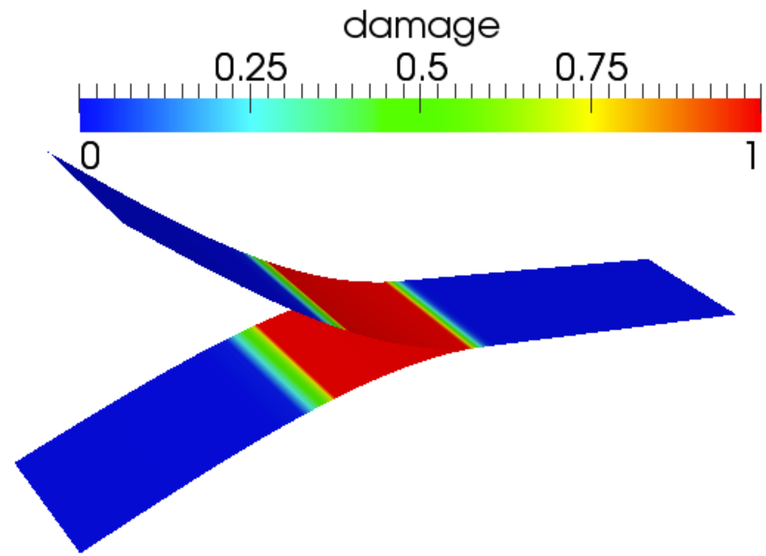
M. A. Gutierrez. Energy release control for numerical simulations of failure in quasi-brittle solids. Communications in Numerical Methods in Engineering, 20(1):19–29, 2004



G. Wimmer and H.E. Pettermann. A semi-analytical model for the simulation of delamination in laminated composites. Composites Science &Technology, 68(12):2332 – 2339, 2008.

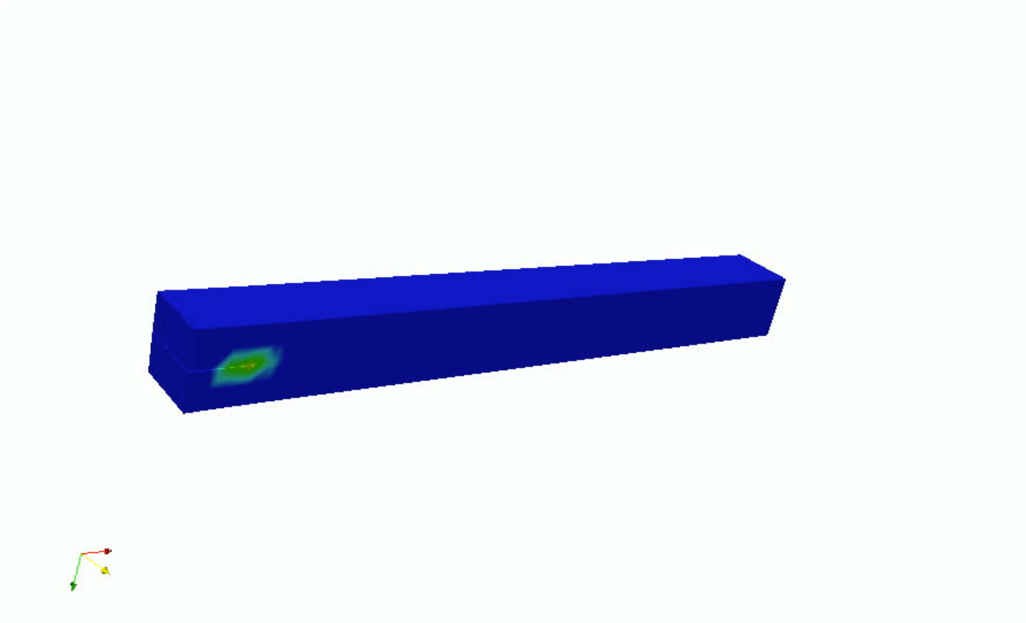
Isogeometric cohesive elements: 3D example with shells

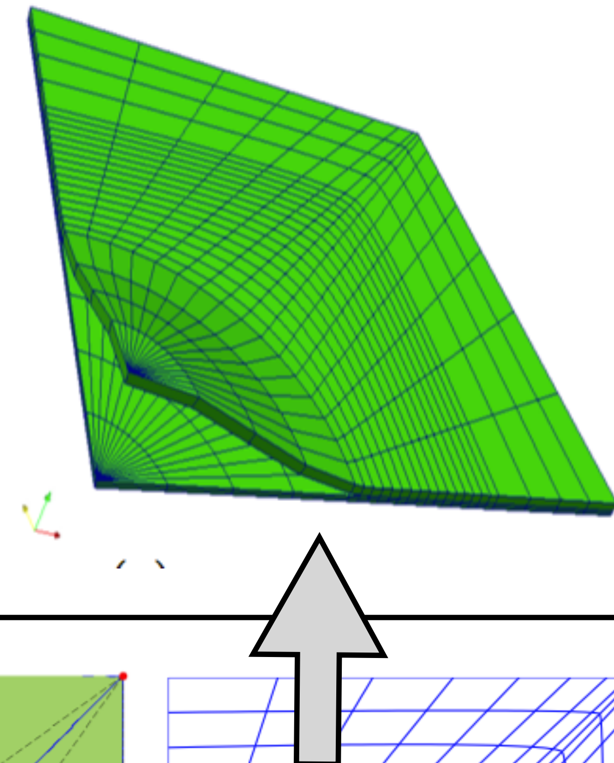




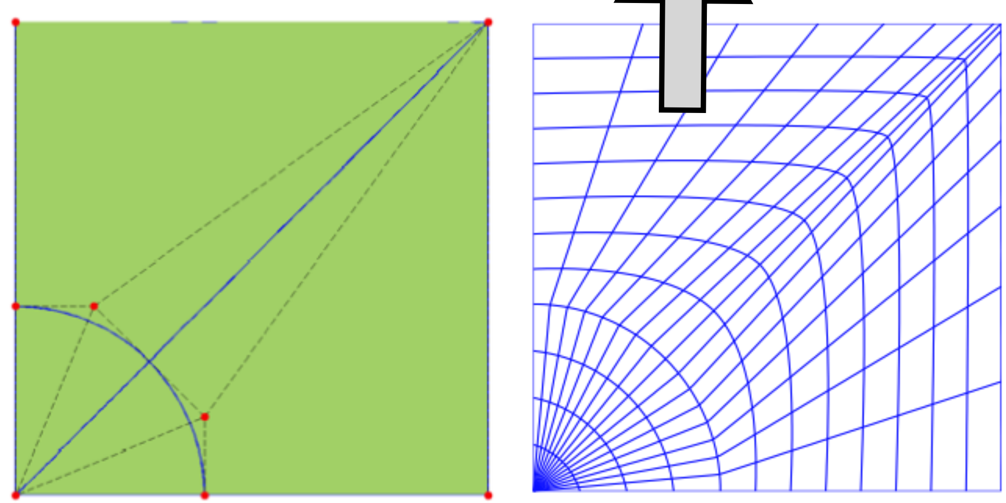
- Rotation free B-splines shell elements (Kiendl et al. CMAME)
- Two shells, one for each lamina
- Bivariate B-splines cohesive interface elements in between

Isogeometric cohesive elements: 3D examples

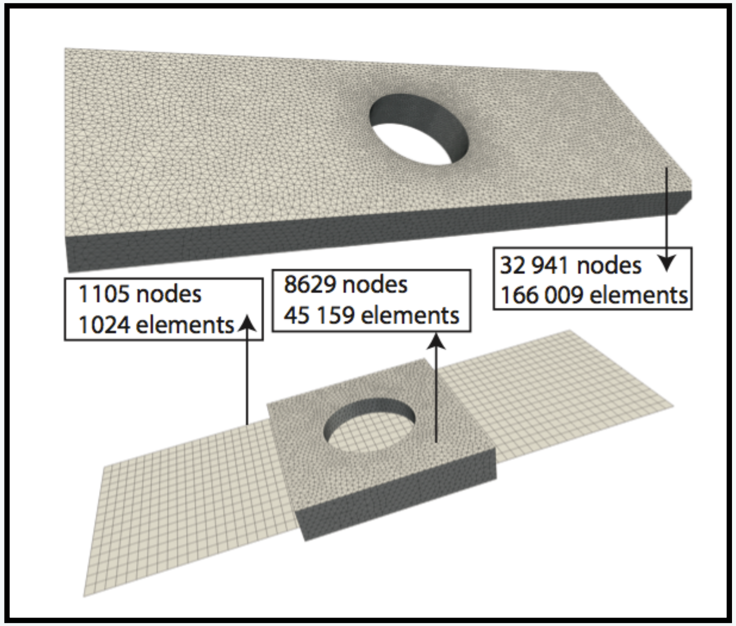


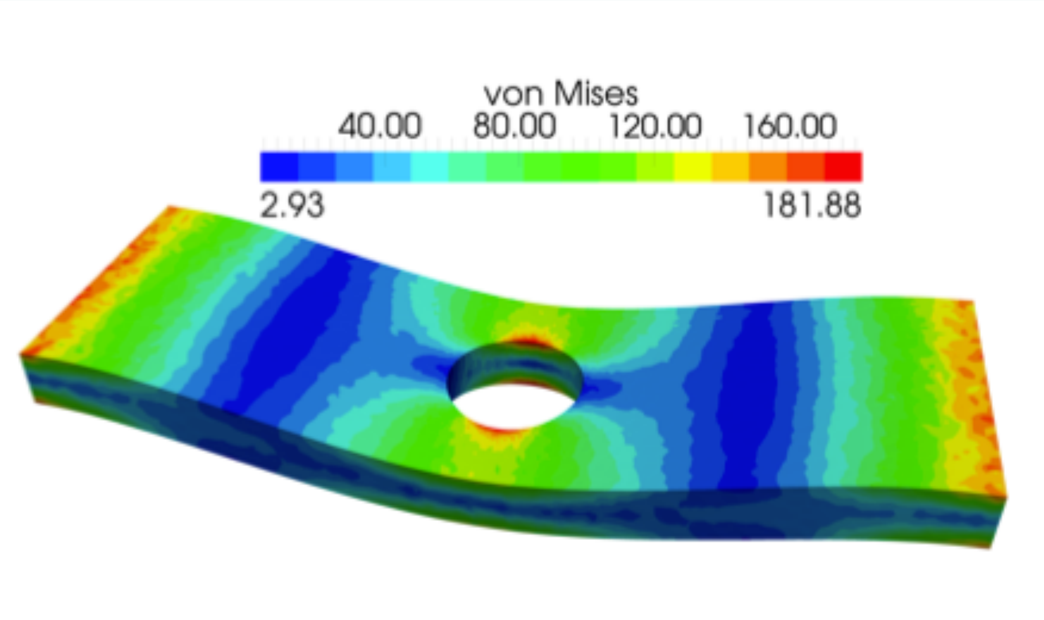


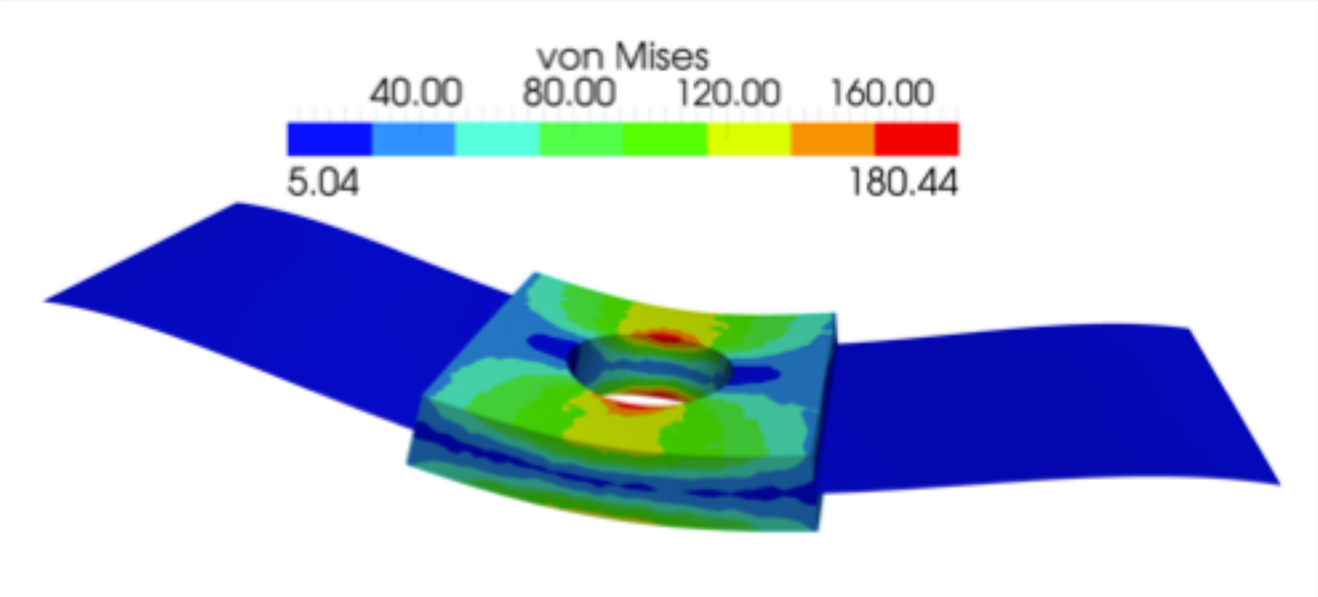
- cohesive elements for 3D meshes the same as 2D
- large deformations
- suitable: delamination buckling analysis



Isogeometric plate and 3D FEM coupling







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Issues

June 2013

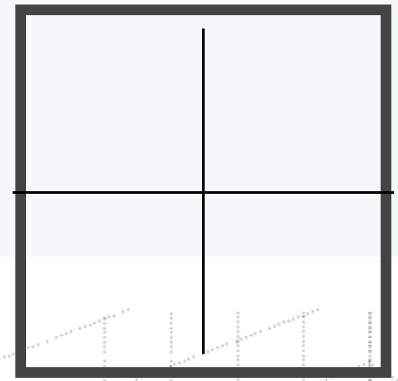
IMAM
Institute of Mechanics & Advanced Materials

Patch

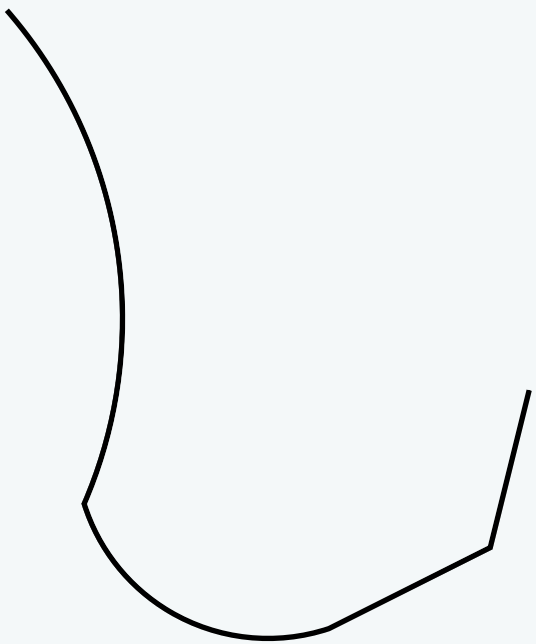
$$\Xi = \{0, 0, 0, 1/2, 1, 1, 1\}$$

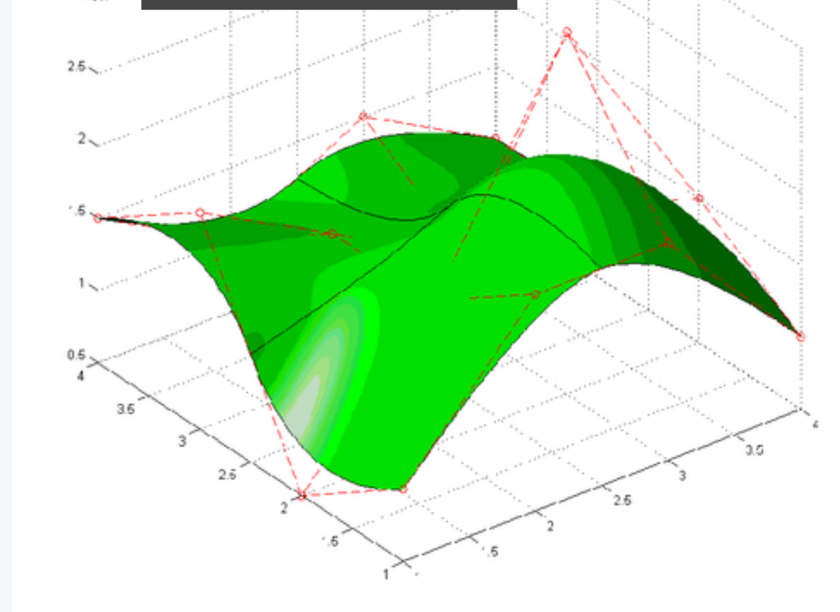
$$\Xi^1 = \{0, 0, 0, 1/2, 1, 1, 1\}$$
$$\Xi^2 = \{0, 0, 0, 1/2, 1, 1, 1\}$$

parameter space

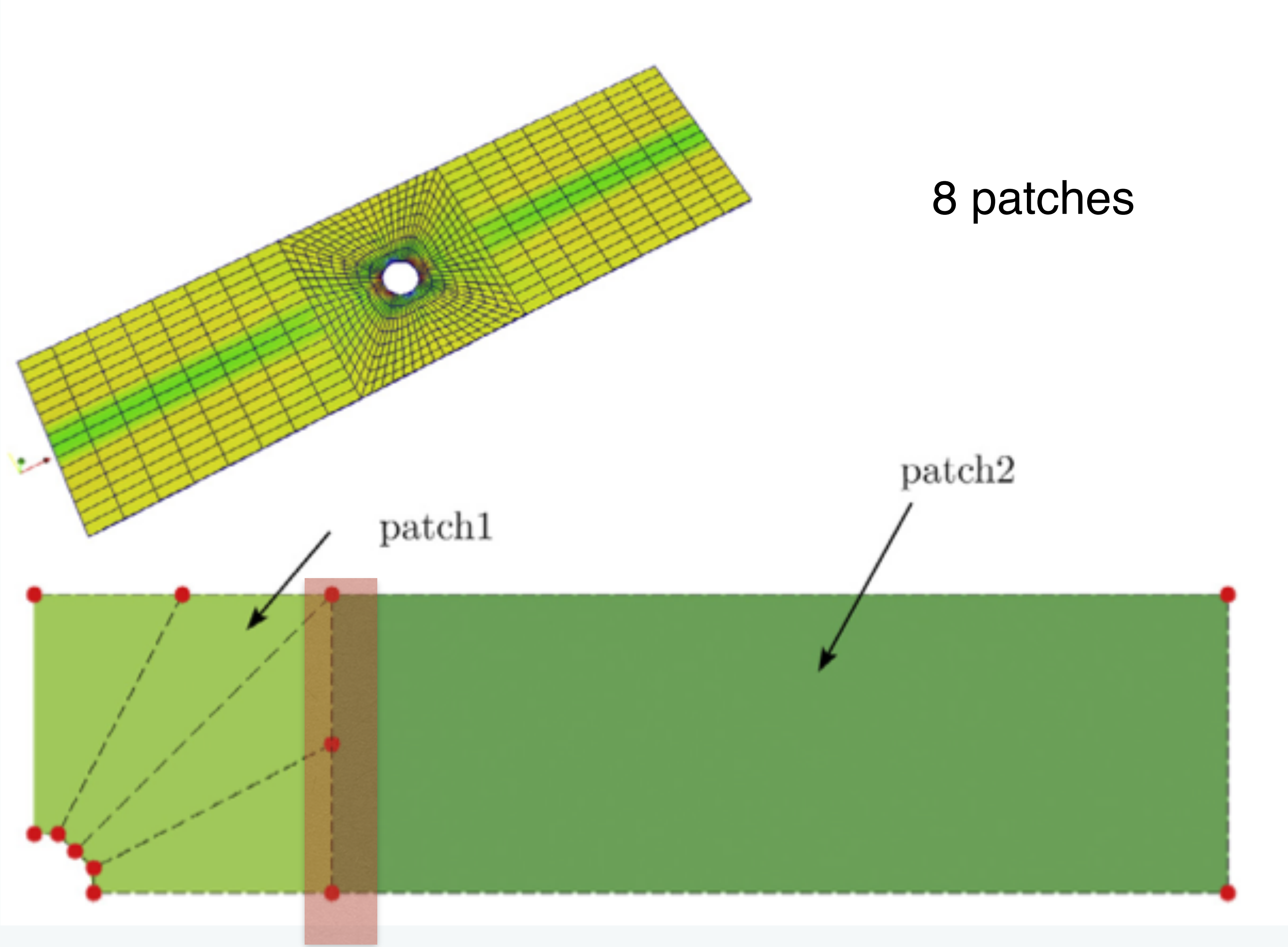


physical space





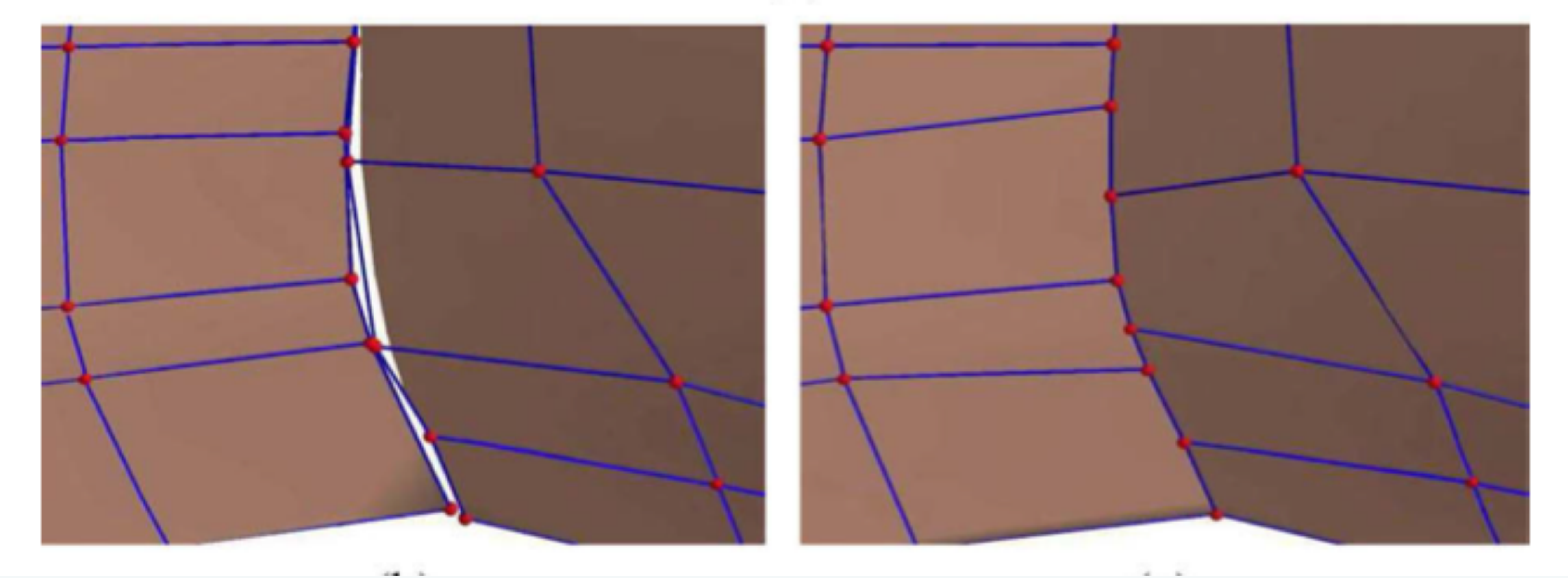
Multipatch NURBS



8 patches

Multipatch NURBS

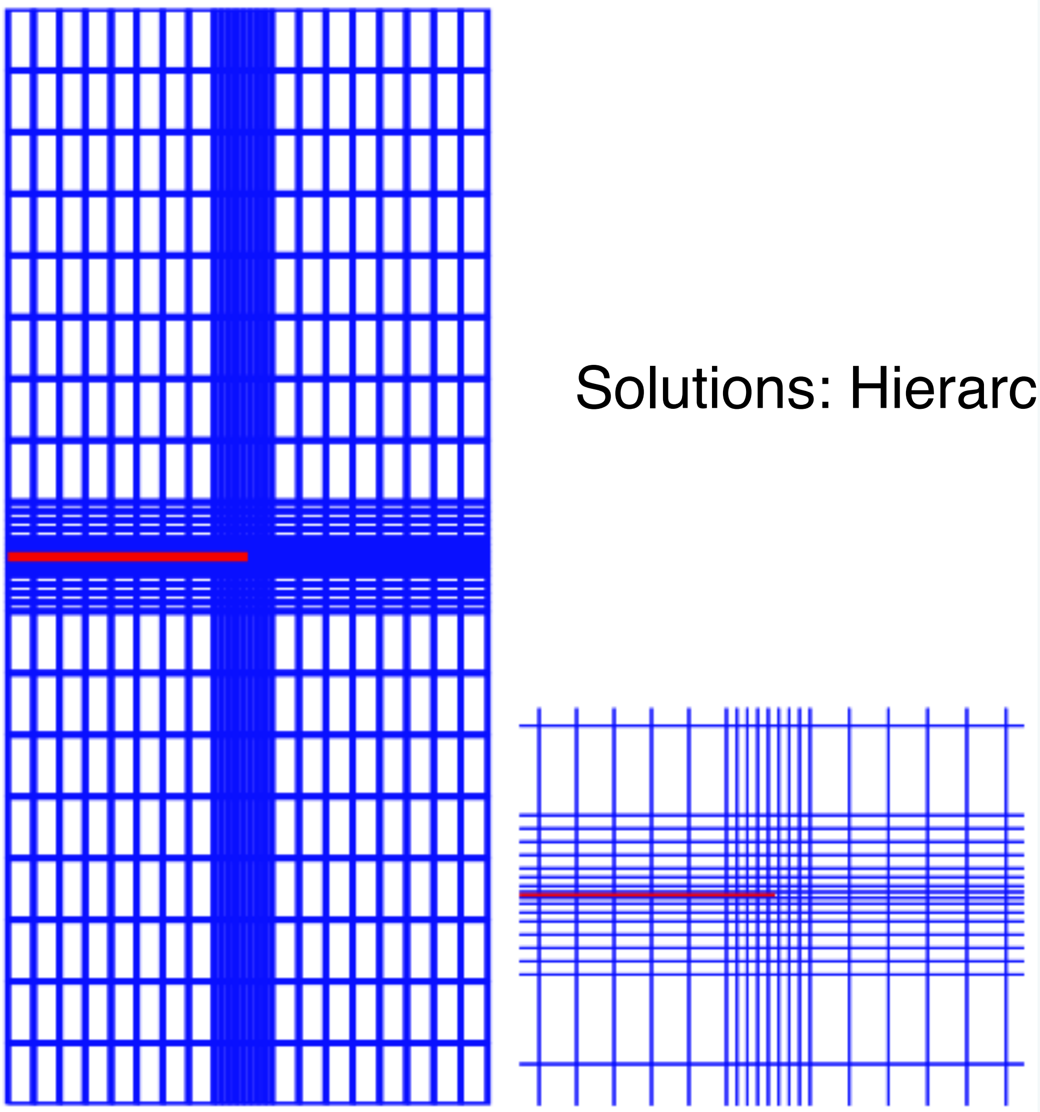
patch2



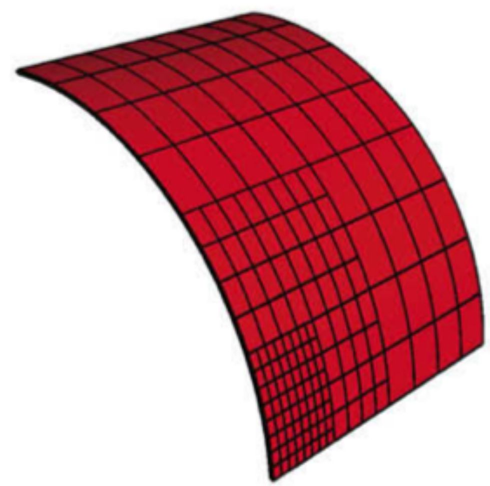
patch1

T-splines

Local refinement



Solutions: Hierarchical B-splines and T-splines



Summary

- B-splines/NURBS/T-splines not only for design but also for analysis
- High order continuity: plate/shell theories, gradient elasticity/damage
- Less prone to locking compared to low-order Lagrange elements
- NURBS of any order achieved with a simple recursion relationship
- Direct link to CAD: optimisation problems
- Smooth geometries: contact problems

- Elaborated enforcement of essential boundary conditions
- Require knowledge on CAD
- More demanding than low order finite elements


Thank You!