Isogeometric cohesive interface elements for 2D/3D delamination analysis

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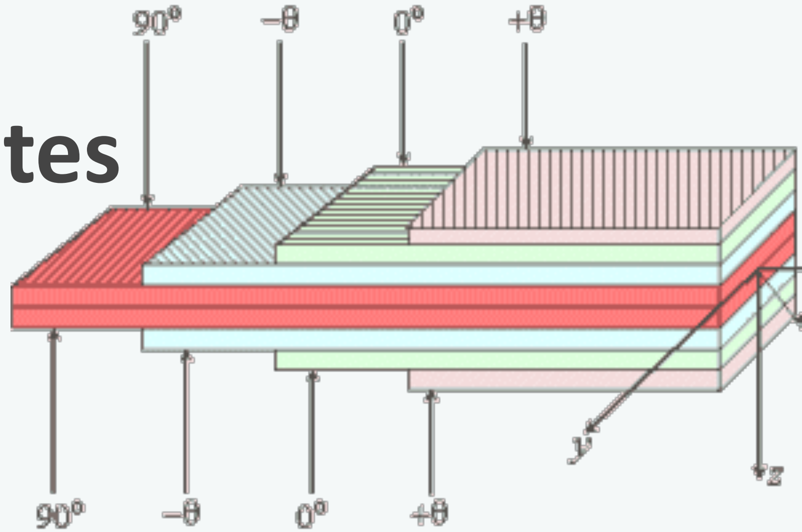
18 June 2013

Failure of composite laminates



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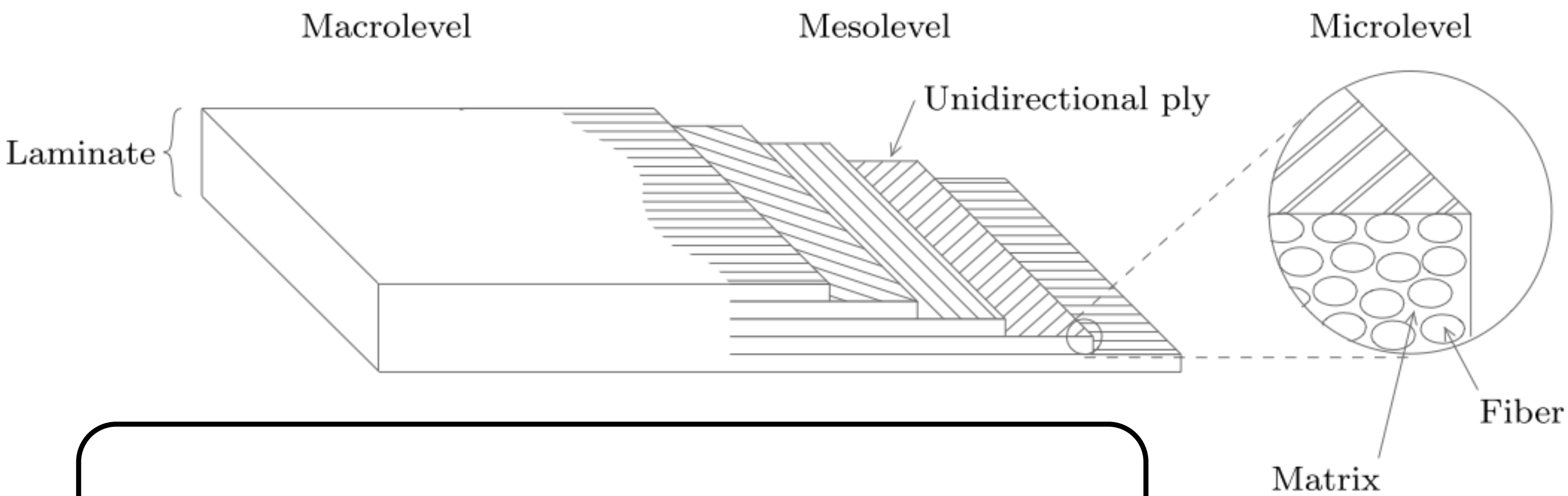
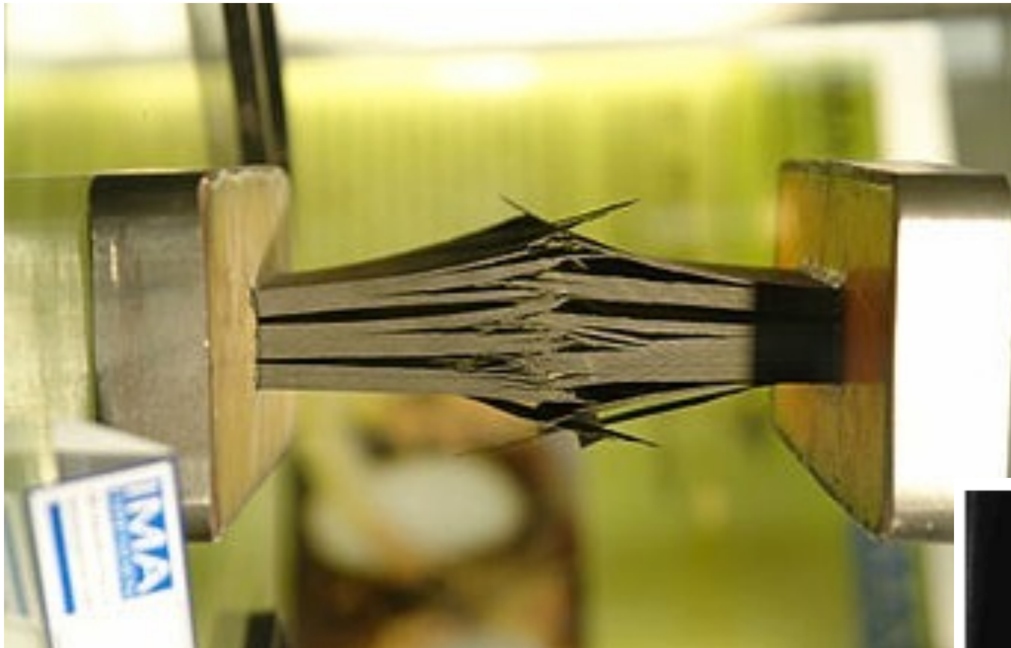
Three levels of observation for composite laminates



- Mesolevel
- Unidirectional ply: orthotropic materials

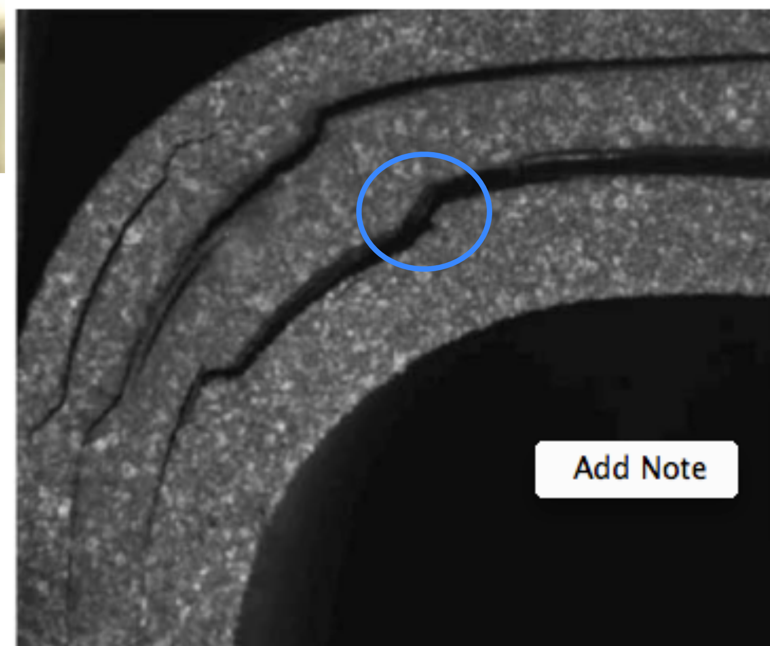
FP van der Meer, Mesolevel Modeling of Failure in Composite Laminates: Constitutive, Kinematic and Algorithmic Aspects, Arch Comput Methods Eng (2012) 19:381–425.

Failure modes of composite laminates



delamination
(interlaminar cracking)

matrix failure
(intralaminar cracking)



Computational modeling of delamination

Finite Element Method (FEM)

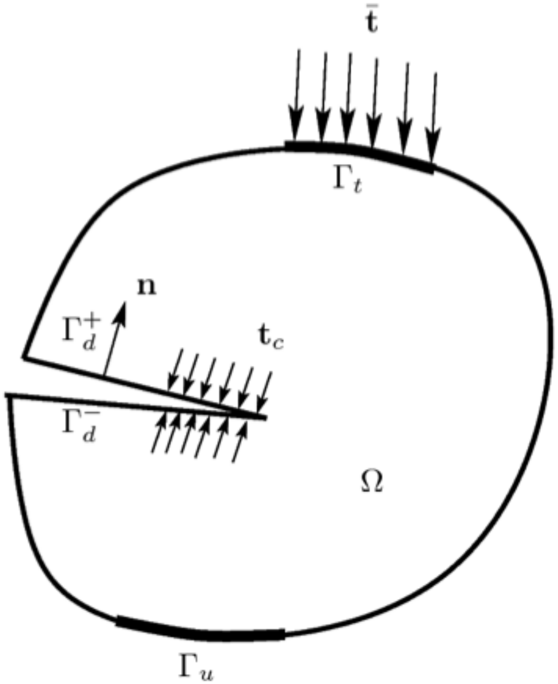
VCCT (Virtual Crack Closure Technique)

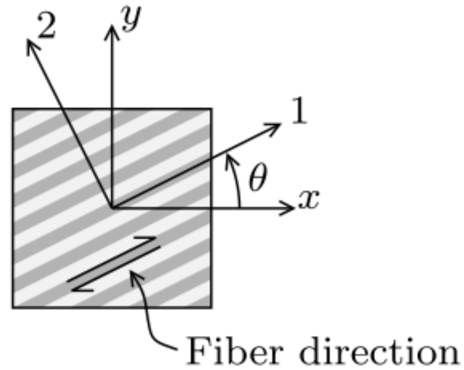
- Linear Elastic Fracture Mechanics
- Only for delamination growth
- Not computationally intensive

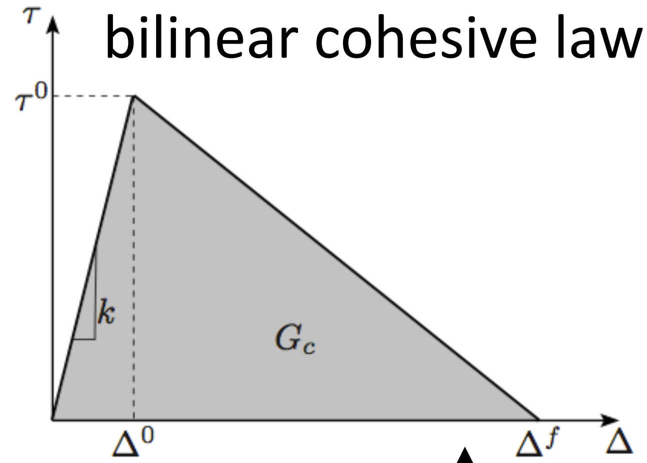
Cohesive interface elements

- Cohesive zone models
- Delamination initiation/growth
- Computationally intensive/robustness issue
- Decohesion elements, cohesive zones, cohesive elements...

Cohesive cracks weak form







$$\bar{\boldsymbol{\sigma}} = \bar{\mathbf{D}}\bar{\boldsymbol{\epsilon}}$$

Unknown field is the displacement u

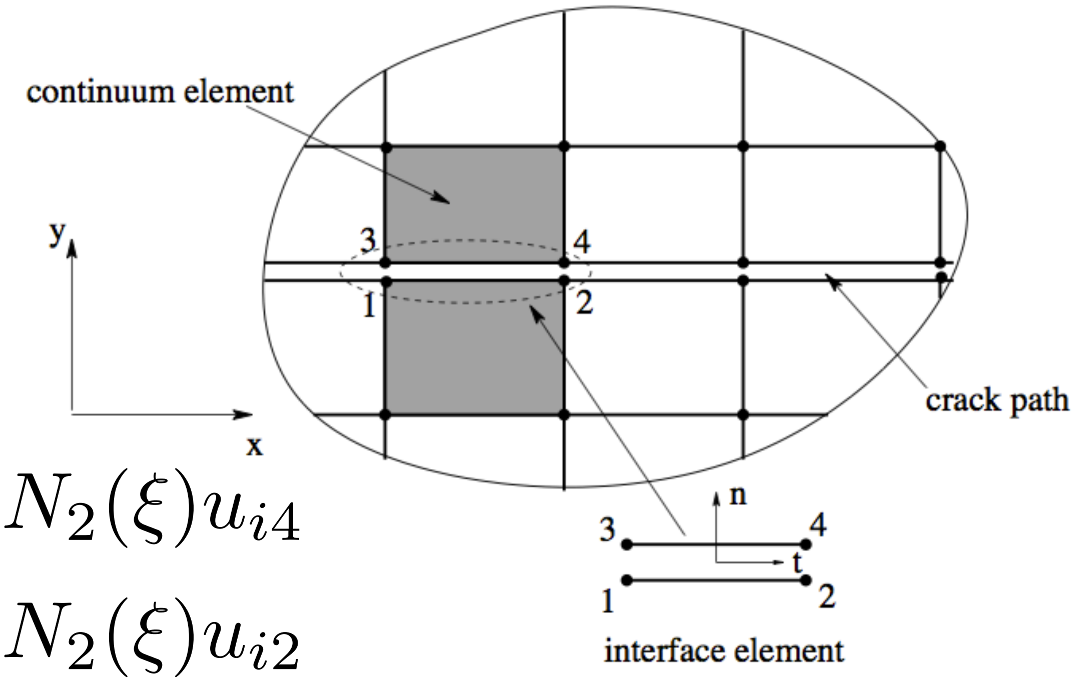
$$\int_\Omega \delta\mathbf{u}\cdot\mathbf{b}\,\mathrm{d}\Omega + \int_{\Gamma_t} \delta\mathbf{u}\cdot\bar{\mathbf{t}}\,\mathrm{d}\Gamma_t = \int_\Omega \delta\boldsymbol{\epsilon} : \boldsymbol{\sigma}(\mathbf{u})\,\mathrm{d}\Omega + \int_{\Gamma_d} \delta[\![\mathbf{u}]\!]\cdot\mathbf{t}^c([\![\mathbf{u}]\!])\,\mathrm{d}\Gamma_d$$

Interface elements

$$\int_{\Omega} \delta \mathbf{u} \cdot \mathbf{b} d\Omega + \int_{\Gamma_t} \delta \mathbf{u} \cdot \bar{\mathbf{t}} d\Gamma_t = \int_{\Omega} \delta \boldsymbol{\epsilon} : \boldsymbol{\sigma}(\mathbf{u}) d\Omega + \int_{\Gamma_d} \delta [\![\mathbf{u}]\!] \cdot \mathbf{t}^{c}([\![\mathbf{u}]\!]) d\Gamma_d$$

Delamination is a problem in which crack path is known in advance.

$$[\![u]\!]_i = u_i^+ - u_i^-$$



$$u_i^+ = N_1(\xi) u_{i3} + N_2(\xi) u_{i4}$$

$$u_i^- = N_1(\xi) u_{i1} + N_2(\xi) u_{i2}$$

Interface elements: internal force vectors

$$\mathbf{f}^{\text{ext}} = \mathbf{f}^{\text{int}} + \mathbf{f}^{\text{coh}}$$

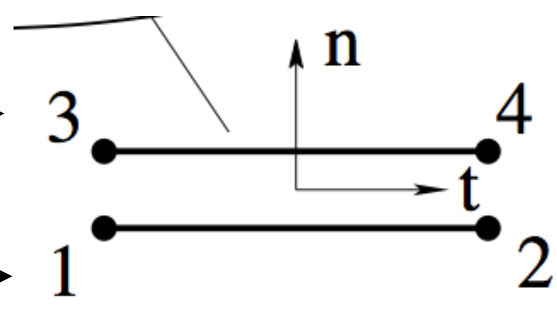
$$\mathbf{f}^{\text{int}} = \int_{\Omega} \mathbf{B}^{\text{T}} \boldsymbol{\sigma} d\Omega$$

$$\mathbf{N}^{\text{int}} = \begin{bmatrix} N_1 & 0 & N_2 & 0 \\ 0 & N_1 & 0 & N_2 \end{bmatrix}$$

$$\mathbf{f}^{\text{ext}} = \int_{\Gamma_t} \mathbf{N}^{\text{T}} \bar{\mathbf{t}} d\Gamma$$

$$\mathbf{f}^{\text{coh}}_{ie,+} = \int_{\Gamma} \mathbf{N}^{\text{T}}_{\text{int}} \mathbf{t}^{\text{c}} \text{d}\Gamma$$

$$\mathbf{f}^{\text{coh}}_{ie,-} = -\int_{\Gamma} \mathbf{N}^{\text{T}}_{\text{int}} \mathbf{t}^{\text{c}} \text{d}\Gamma$$

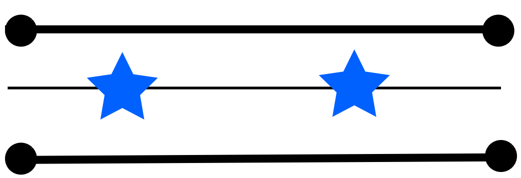


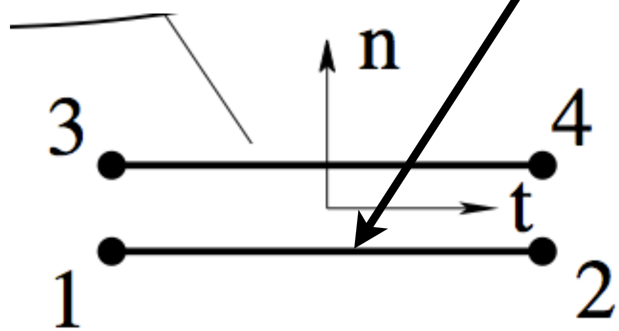
interface element

Interface elements: tangent matrix

$$\mathbf{K}_e^{int} = \begin{bmatrix} \int_\Gamma \mathbf{N}^{\mathrm{T}}\mathbf{Q}^{\mathrm{T}}\mathbf{T}\mathbf{Q}\mathbf{N}d\Gamma & -\int_\Gamma \mathbf{N}^{\mathrm{T}}\mathbf{Q}^{\mathrm{T}}\mathbf{T}\mathbf{Q}\mathbf{N}d\Gamma \\ -\int_\Gamma \mathbf{N}^{\mathrm{T}}\mathbf{Q}^{\mathrm{T}}\mathbf{T}\mathbf{Q}\mathbf{N}d\Gamma & \int_\Gamma \mathbf{N}^{\mathrm{T}}\mathbf{Q}^{\mathrm{T}}\mathbf{T}\mathbf{Q}\mathbf{N}d\Gamma \end{bmatrix}$$

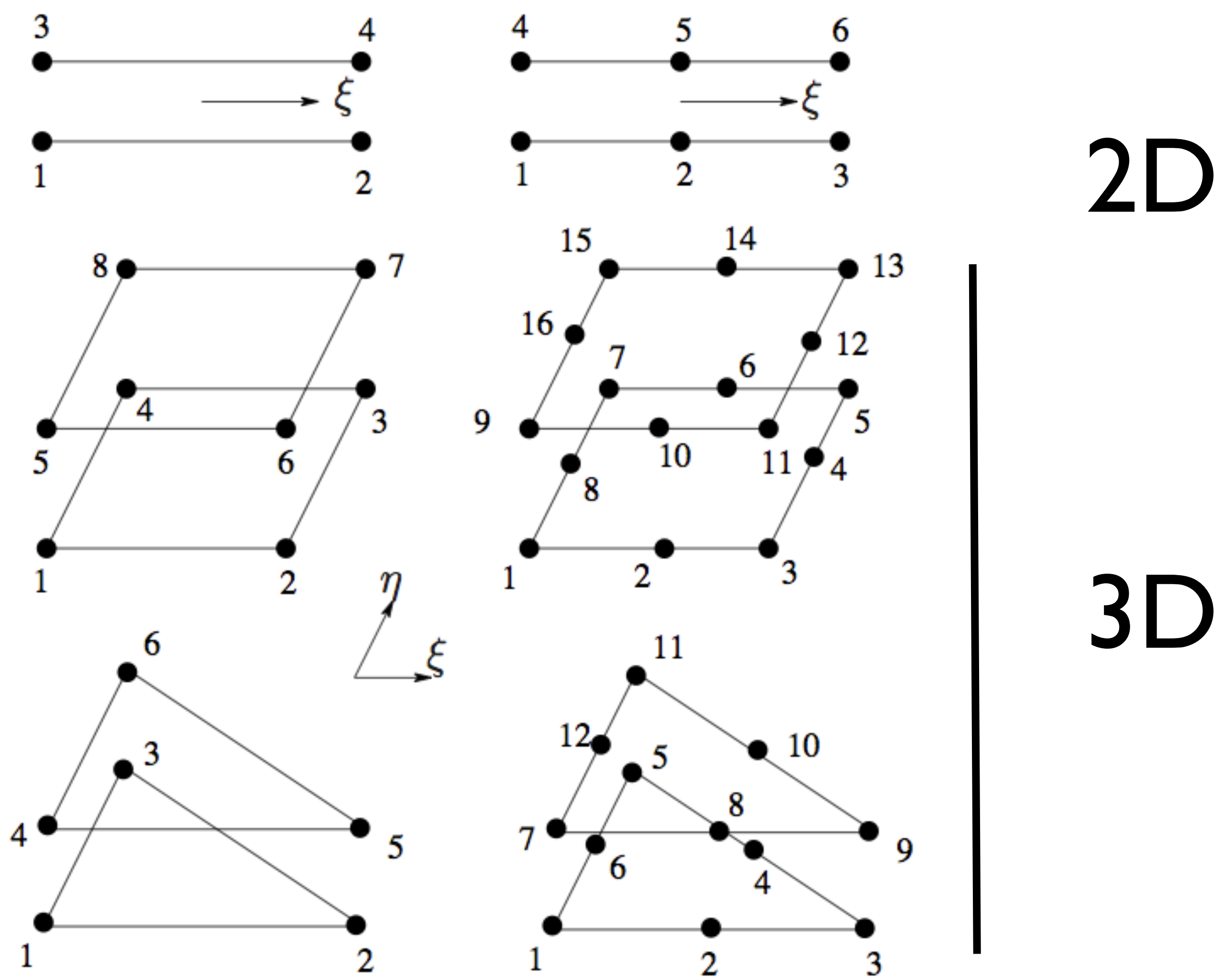
Numerical integration





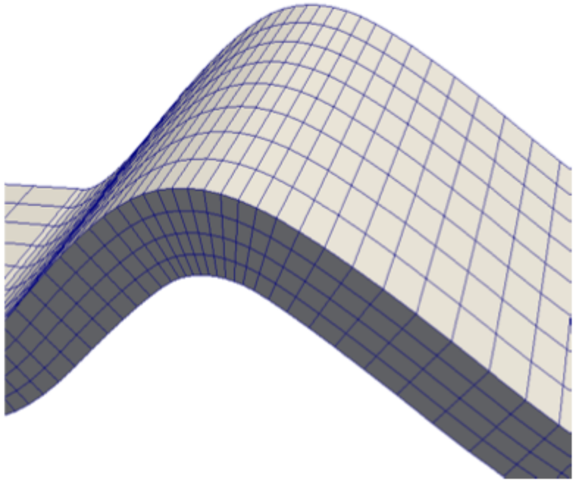
interface element

Common interface elements



What is wrong with standard interface elements?

- time consuming pre-processing (generation of interface el.)
- no link to CAD data: not ideal for design-analysis cycles
- standard low order Lagrange elements: poor derivative fields such as stresses => very fine mesh in front of the crack tip
- geometry: not exactly represented



Isogeometric interface elements

- fast pre-processing: automatic generation of interface el.
- link to CAD data: ideal for design-analysis cycles
- high order NURBS elements: highly accurate derivative fields
- less expensive than low order Lagrange elements
- geometry: exactly represented

There are no free lunch. However let talk about the good news first.

Isogeometric analysis

June 2013

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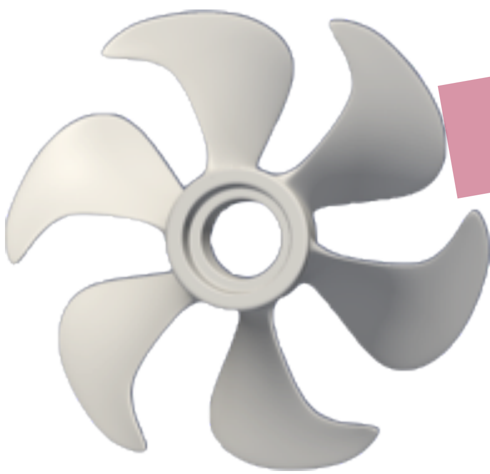
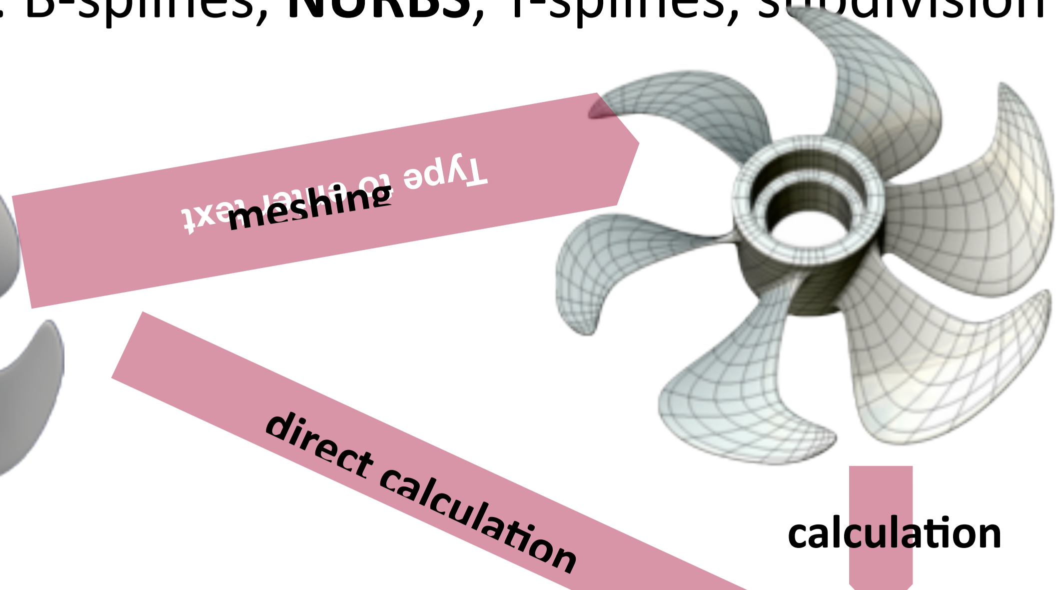
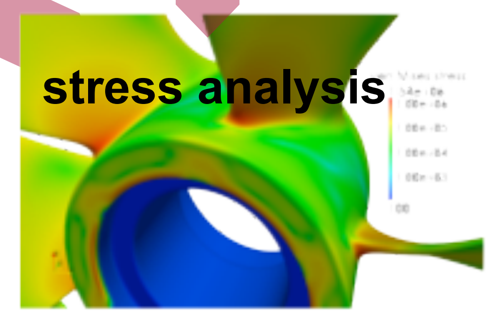
Isogeometric analysis

Approximate the unknown fields with the basis functions used to generate the CAD model.

CAD basis functions: B-splines, **NURBS**, T-splines, subdivision surfaces...







- Exact geometry
- High order continuity
- *hpk*-refinement

MA Scott et al, CMAME 2013.

CAD-FEA integration: literature review

- P. Kagan, A. Fischer, and P. Z. Bar-Yoseph. New B-Spline Finite Element approach for geometrical design and mechanical analysis. IJNME, 41(3):435–458, 1998.
- F. Cirak, M. Ortiz, and P. Schroder. Subdivision surfaces: a new paradigm for thin-shell finite-element analysis. IJNME, 47(12): 2039–2072, 2000.
- Constructive solid analysis: a hierarchical, geometry-based meshless analysis procedure for integrated design and analysis. D. Natekar, S. Zhang,and G. Subbarayan. CAD, 36(5): 473--486, 2004.
- T.J.R. Hughes, J.A. Cottrell, and Y. Bazilevs. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. CMAME, 194(39-41):4135–4195, 2005.
- J. A. Cottrell, T. J.R. Hughes, and Y. Bazilevs. Isogeometric Analysis: Toward Integration of CAD and FEA. Wiley, 2009.

B-splines basis functions

$$\Xi = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}$$ knot vector

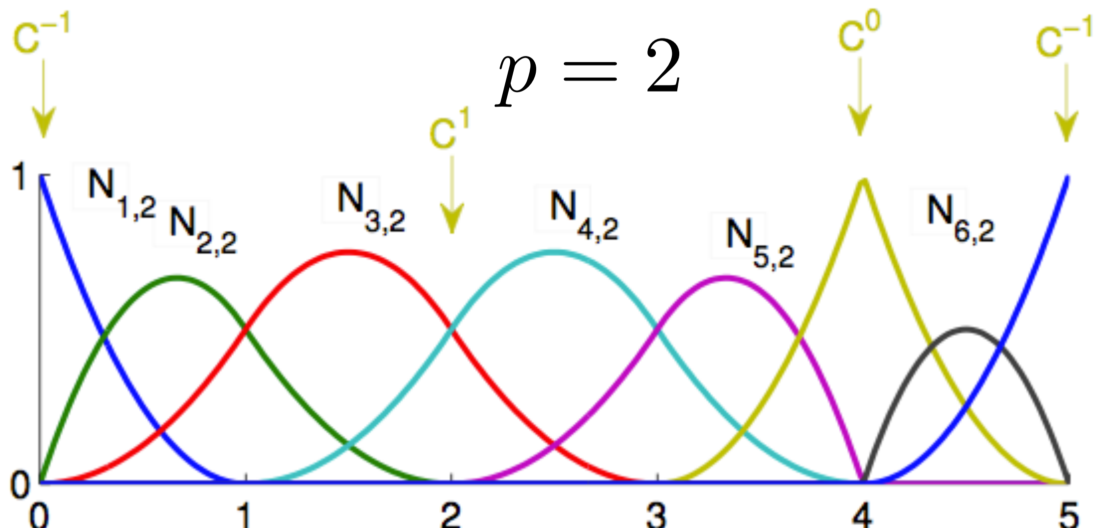
$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \le \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

\sigma

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$

Properties

- Partition of Unity
- Linear independence
- Non-negativity
- C^{p-m} continuity
- Not interpolants



$$\Xi = \{0, 0, 0, 1, 2, 3, 4, 4, 5, 5, 5\}$$

B-splines

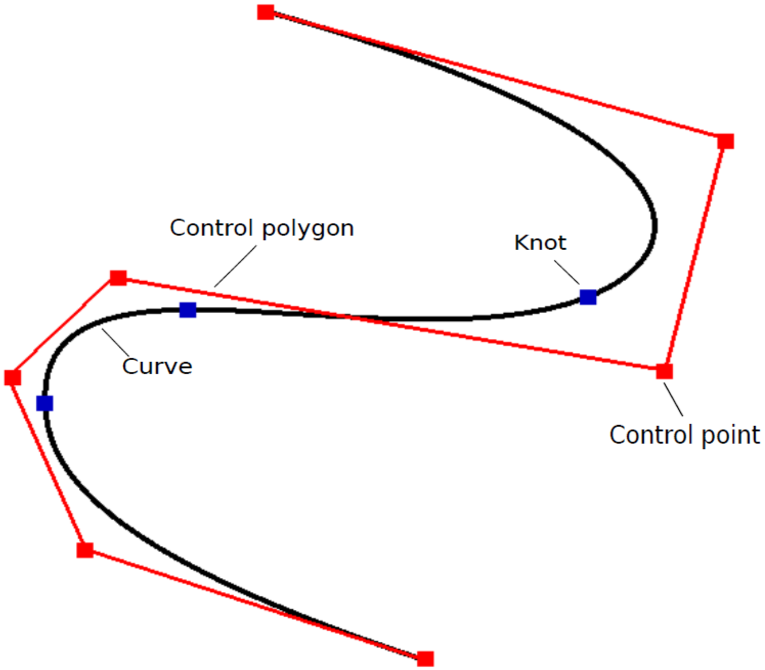
$$\Xi = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}$$

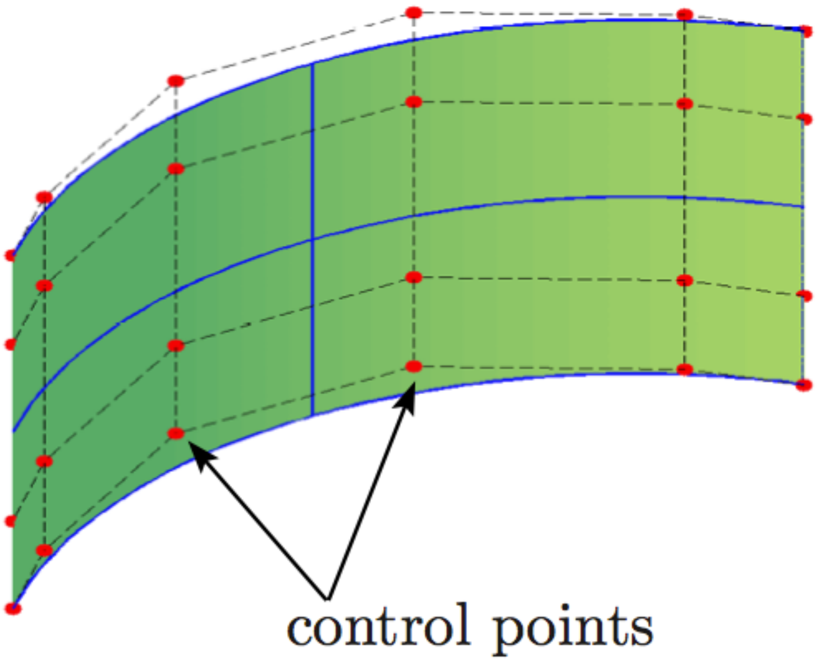
$$\Xi^1 = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}$$

$$\Xi^2 = \{\eta_1, \eta_2, \ldots, \eta_{m+q+1}\}$$

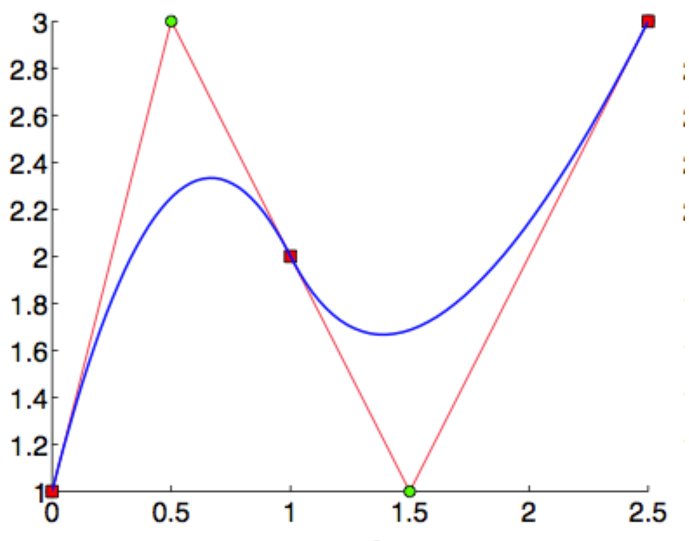
$$\mathbf{C}(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi)\mathbf{B}_i$$

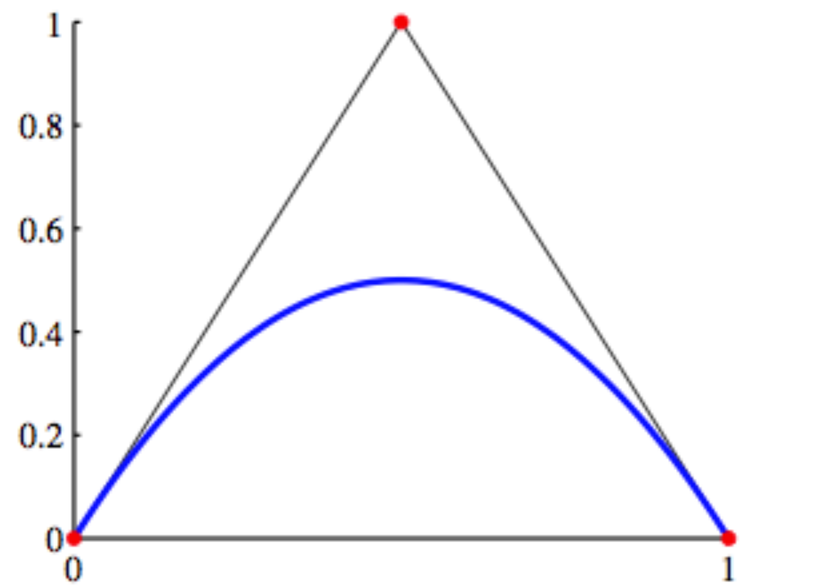
$$\mathbf{S}(\xi, \eta) = \sum_{i=1}^{n}\sum_{j=1}^{m} N_{i,p}(\xi)M_{j,q}(\eta)\mathbf{B}_{ij}$$



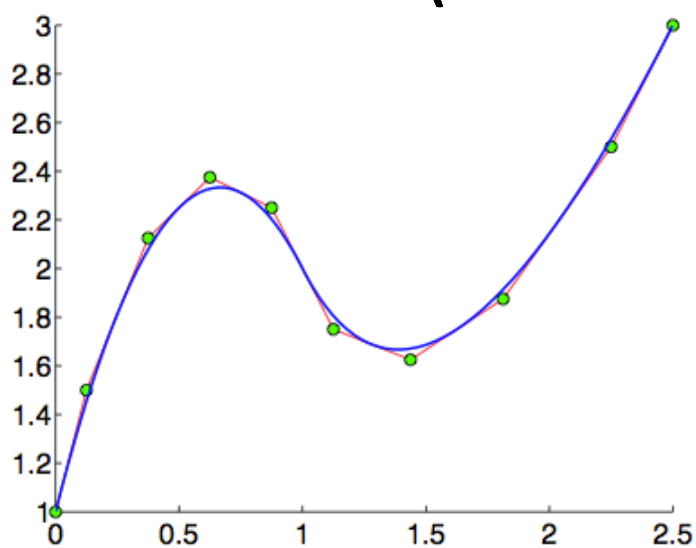


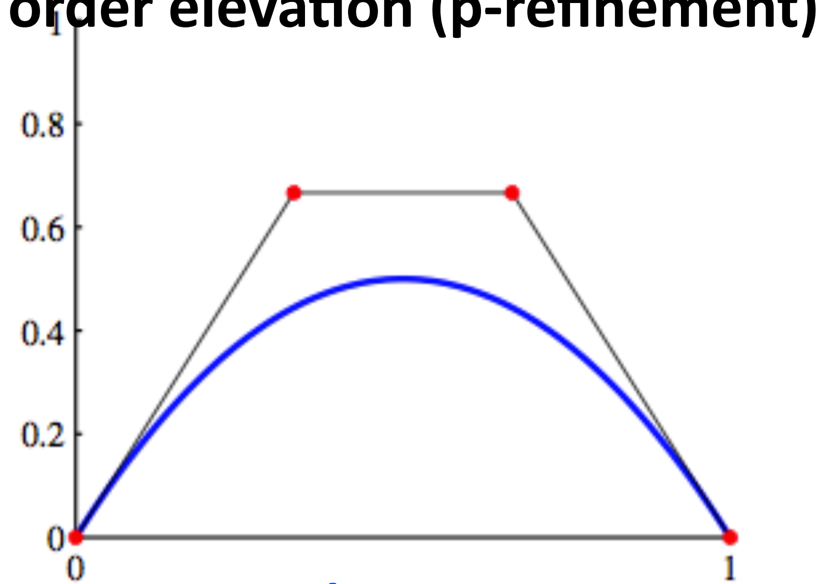
Enriching B-splines





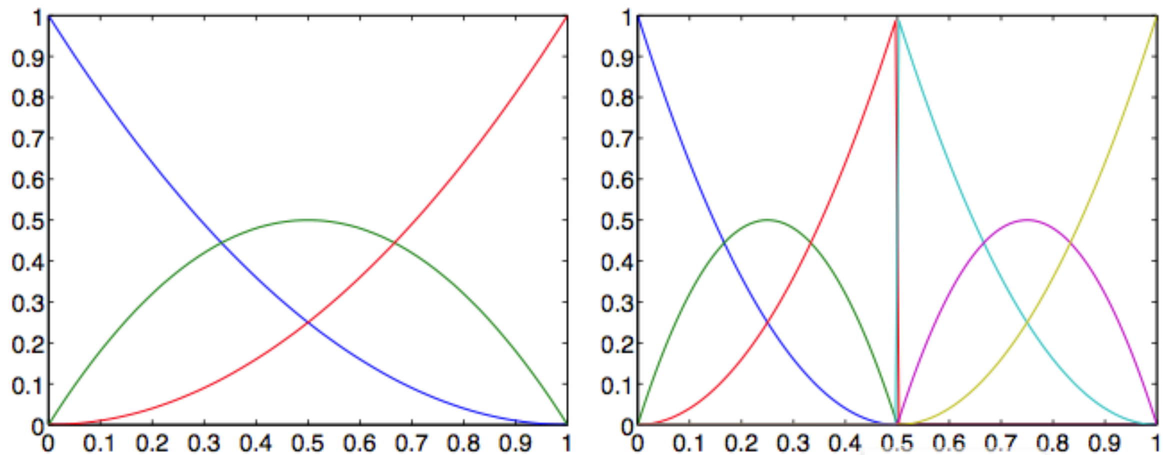
knot insertion (h-refinement) + order elevation (p-refinement)



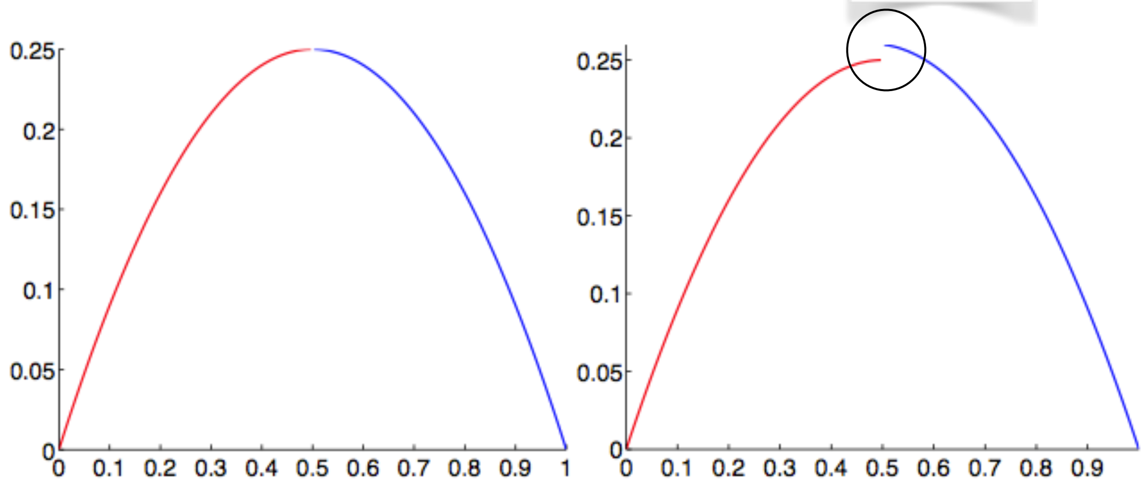


does not change B-splines geometrically/parametrically

Knot insertion to create discontinuities



(a) $\Xi = \{0, 0, 0, 1, 1, 1\}$ (b) $\Xi' = \{0, 0, 0, 0.5, 0.5, 0.5, 1, 1, 1\}$

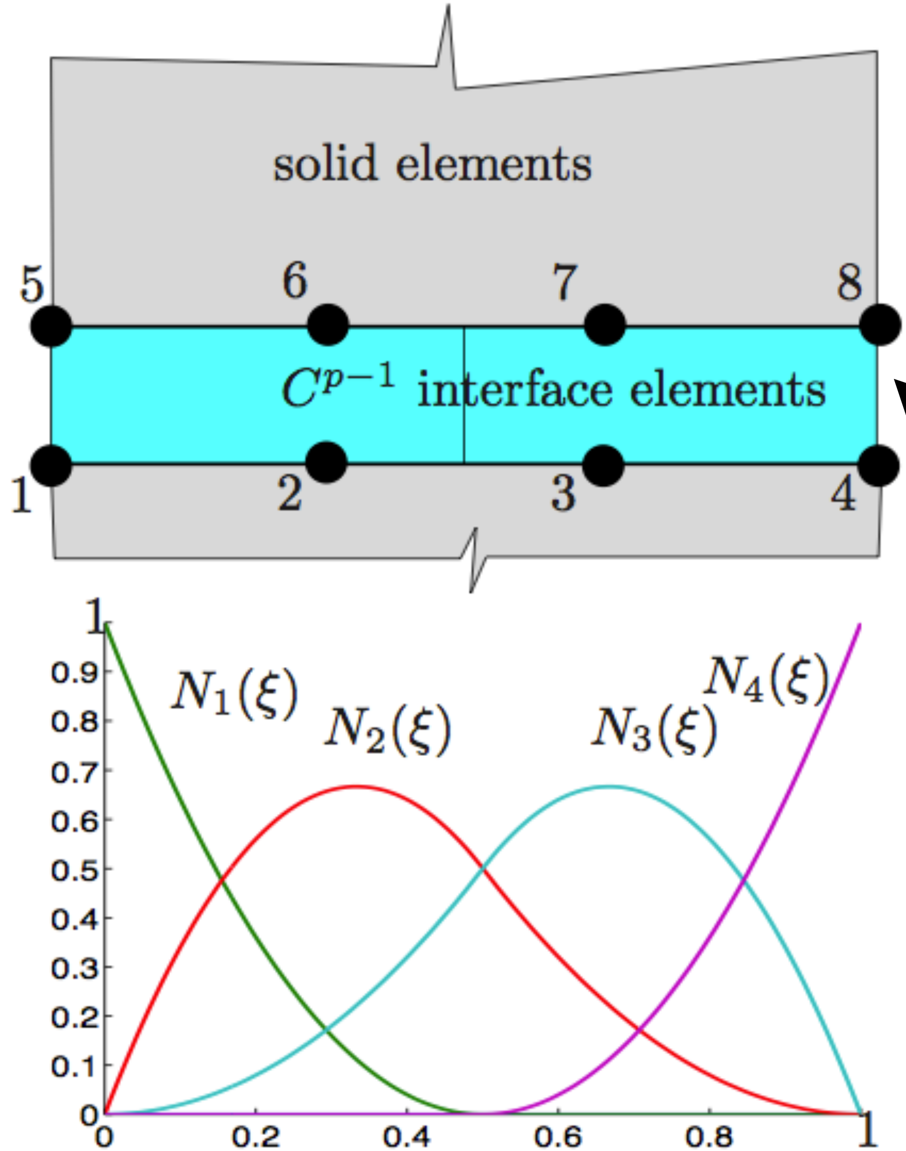


$$p = 2$$

does not change B-splines geometrically/parametrically

knot insertion: stable algorithms, available implementations

Isogeometric cohesive elements



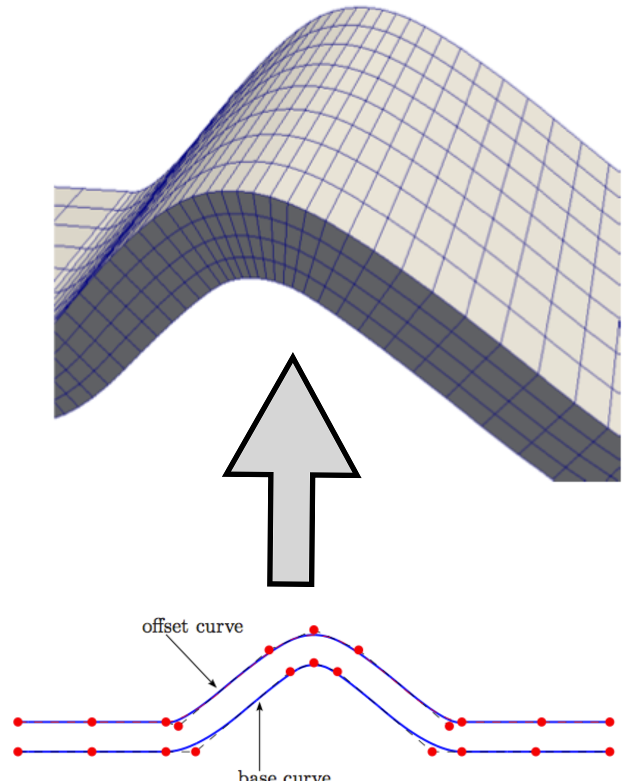
2 quadratic int. elements

[1,2,3,5,6,7]
[2,3,4,6,7,8]

automatically generated using knot insertion.

Isogeometric cohesive elements: advantages

- 2D Mixed mode bending test (MMB)
- 2 x 70 quartic-linear B-spline elements
- run time on a laptop 4GB of RAM: 6 s
- energy arc-length control



1. C. V. Verhoosel, M. A. Scott, R. de Borst, and T. J. R. Hughes. An isogeometric approach to cohesive zone modeling. IJNME, 87:336–360, 2011.
2. V. P. Nguyen and H. Nguyen-Xuan. High-order B-splines based finite elements for delamination analysis of laminated composites. Com. Str., 102:261–275, 2013.
3. V.P. Nguyen, P. Kerfriden, S. Bordas. Isogeometric cohesive elements for two and three dimensional composite delamination analysis, 2013, Arxiv.

Examples

Tools

MIGFEM

- quick prototyping
- tutorial codes

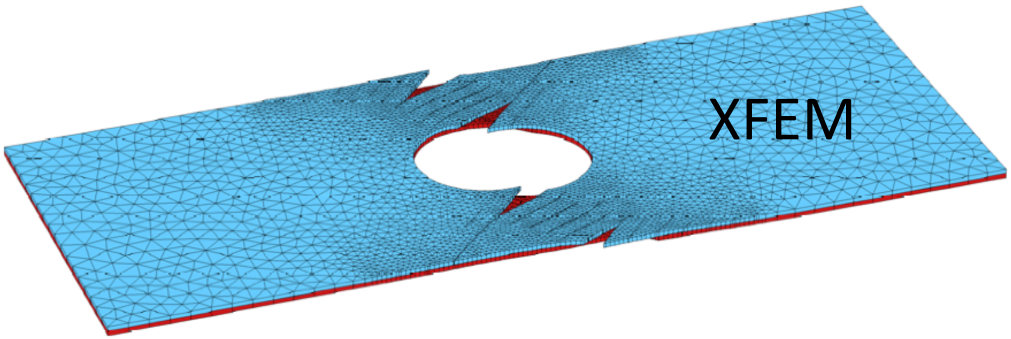
- open source Matlab Isogeometric (X)FEM
- 2D/3D solid mechanics with geometry nonlinearities
- 2D XIGA for LEFM and material interfaces
- Structural mechanics: beam, plate, shells (large deformation)
- http://sourceforge.net/projects/cmcodes/

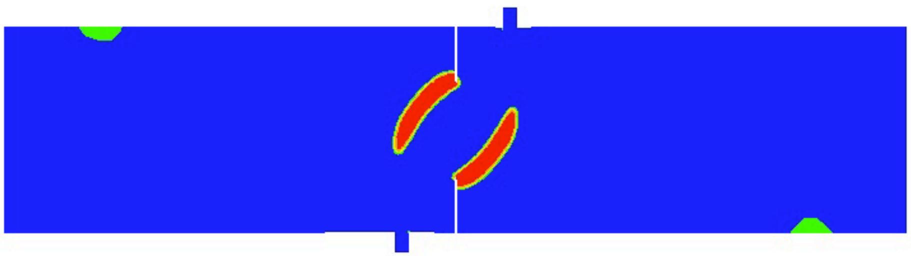
jem-jive (Linux, Mac OS, Windows)

DRG Dynaflow Research Group

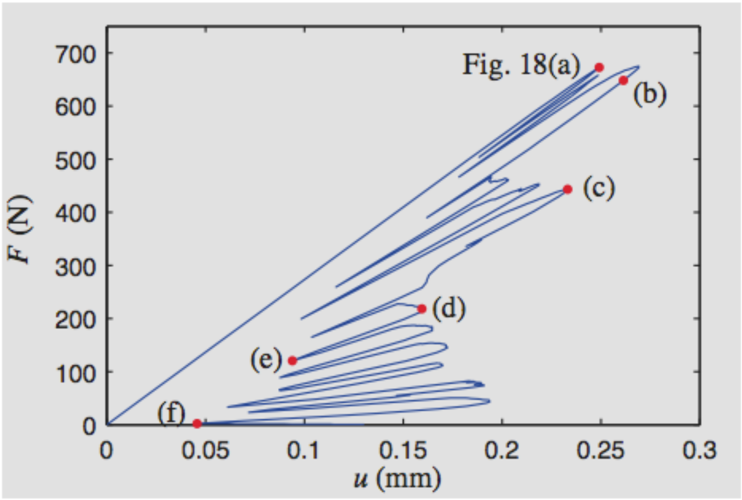
- commericial C++ toolkit for PDEs
- not a general purpose FE package
- tailor made applications, suitable for researchers
- apps: XFEM, dG, IGA, DEM, FVM etc.
- support parallel computing
- implements useful concepts available in other programming languages--Java, Fortran 90, Matlab and C#
- tensor class: useful to evaluating complex constitutive models
- http://www.dynaflow.com/en_GB/jive.html

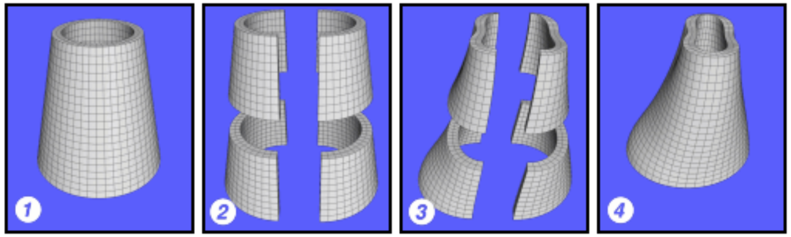
jem-jive: some typical examples

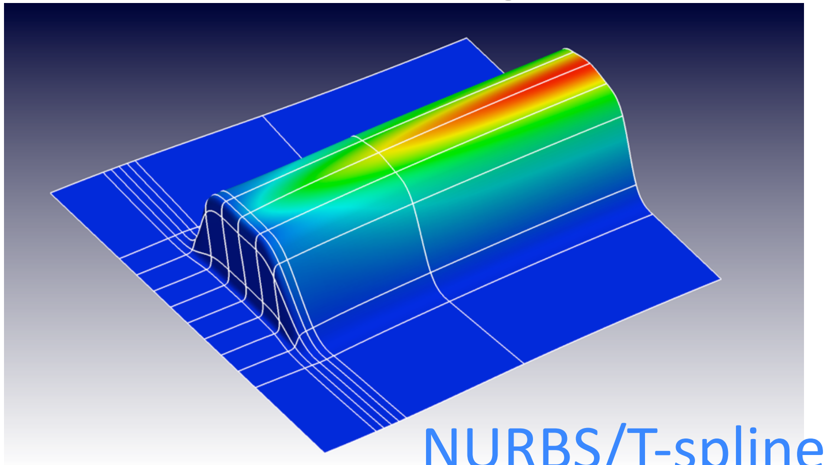




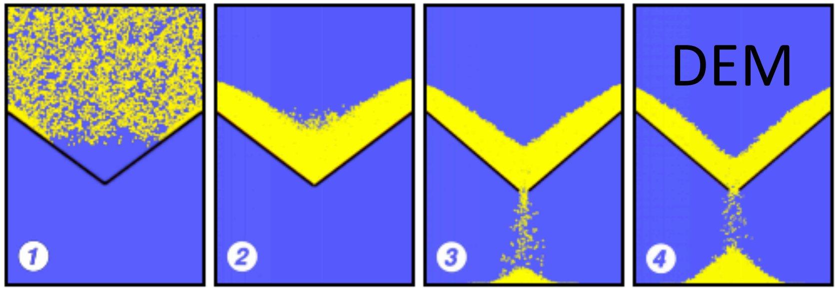
non-local damage model



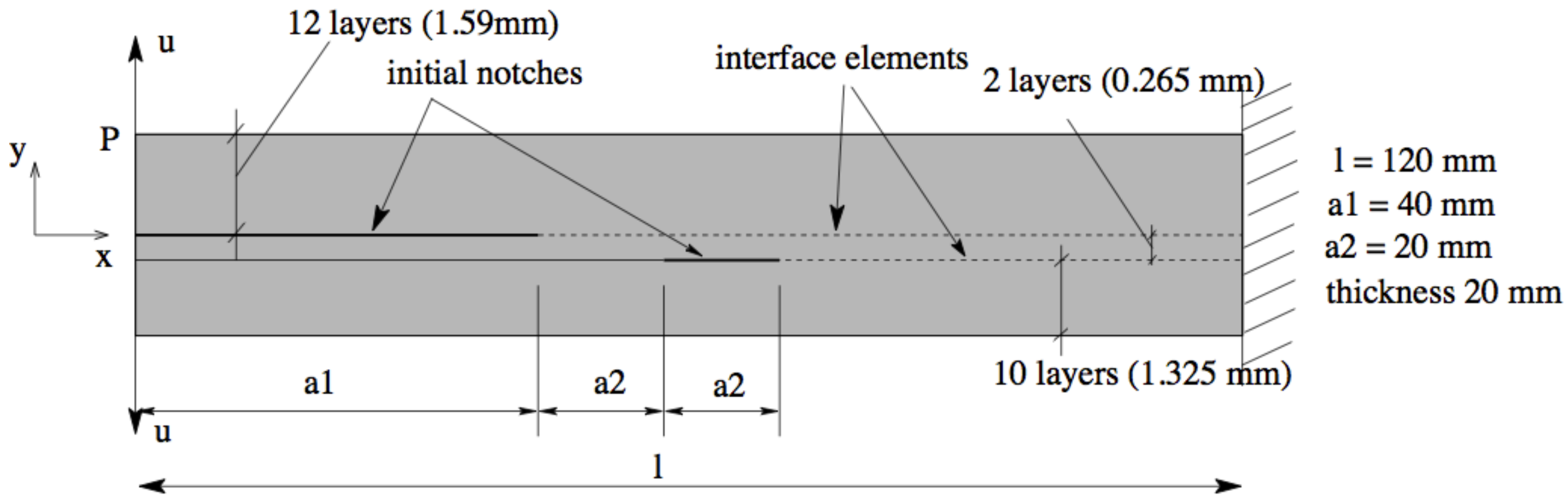


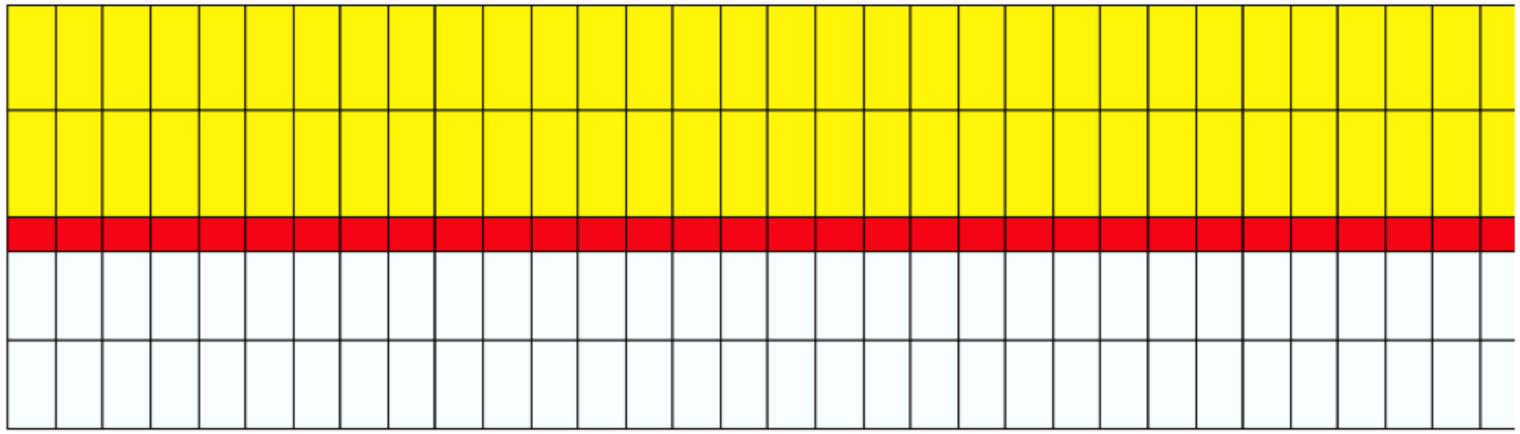


NURBS/T-splines

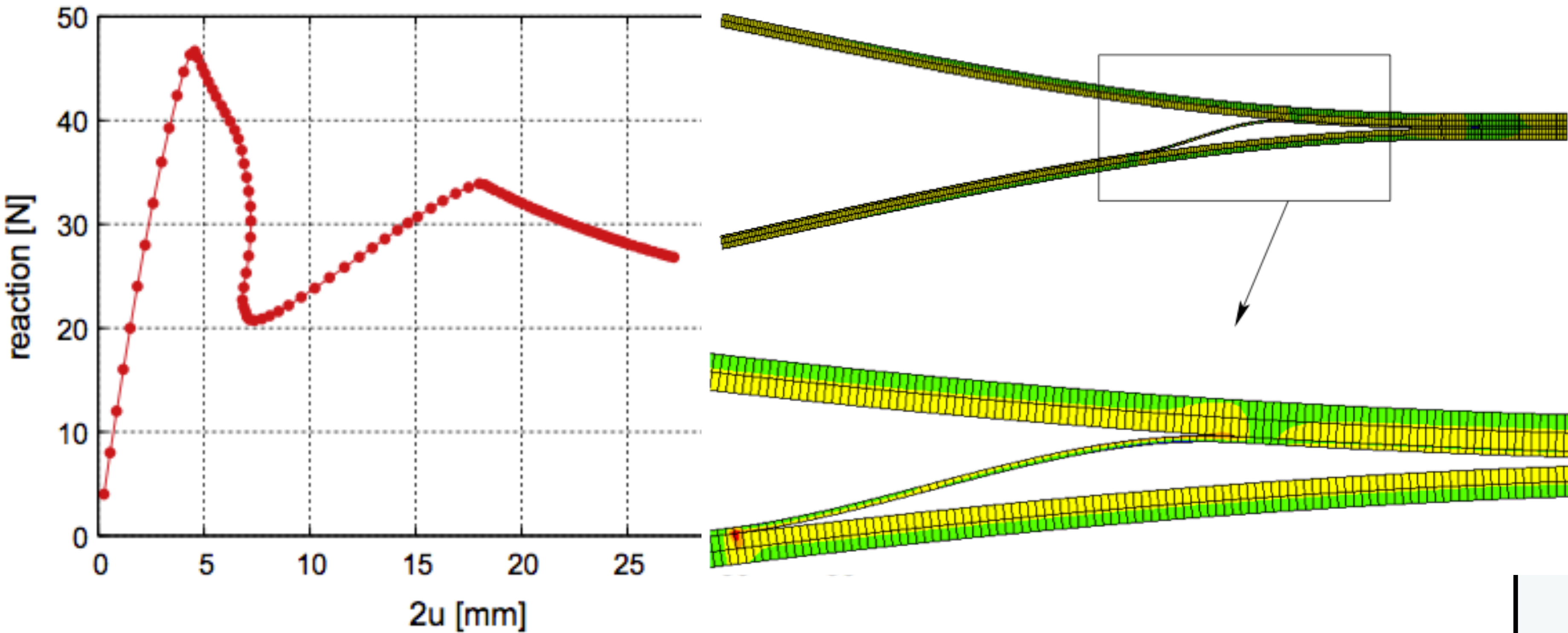


Isogeometric cohesive elements: 2D example





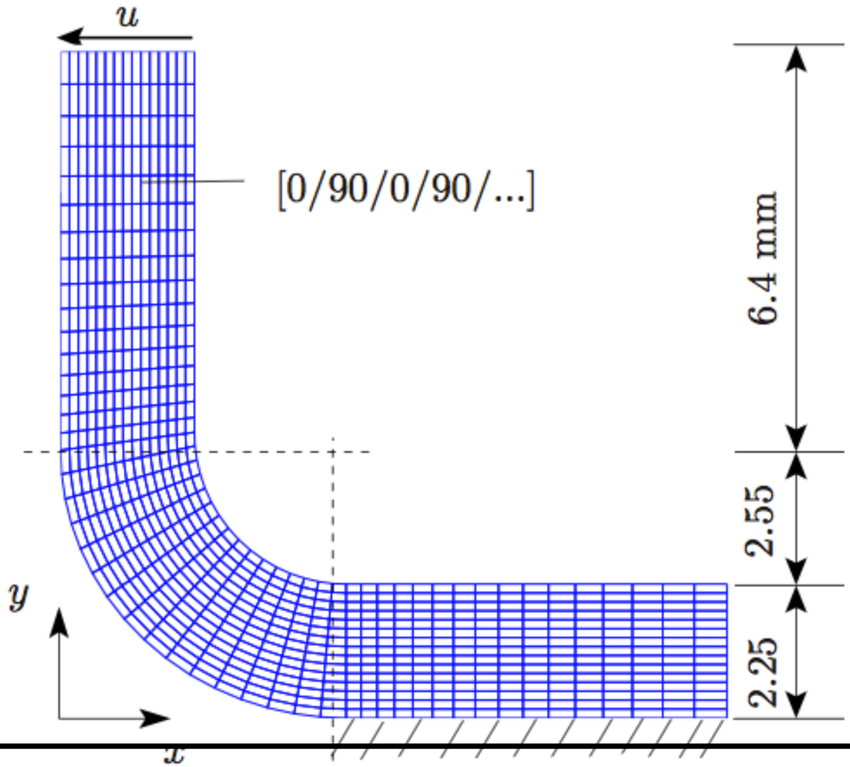
Isogeometric cohesive elements: 2D example

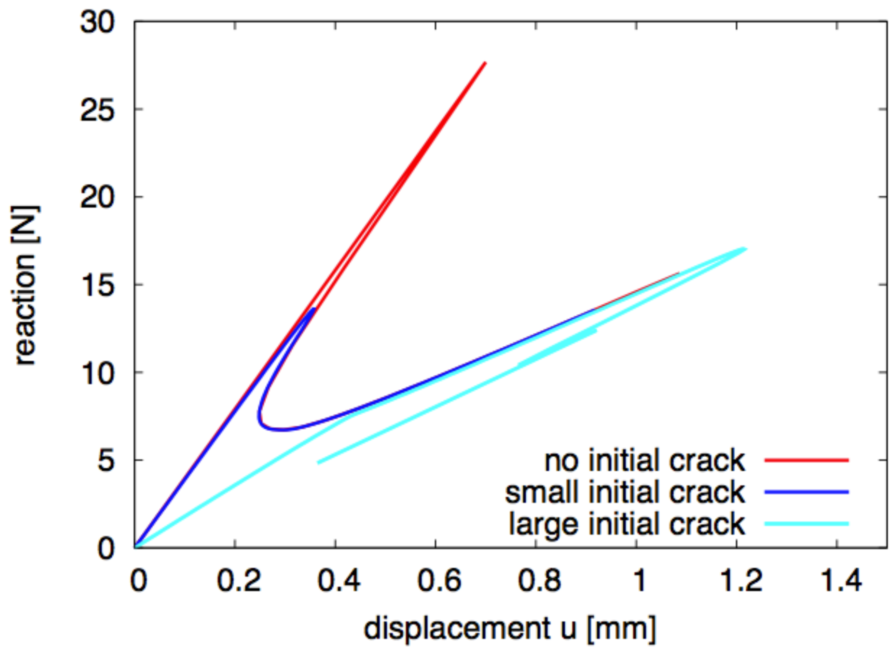


load-displacement response

Alfano G, Crisfield MA. Finite element interface models for the delamination analysis of laminated composites: mechanical and computational issues. IJNME 2001;50(7):1701–36.

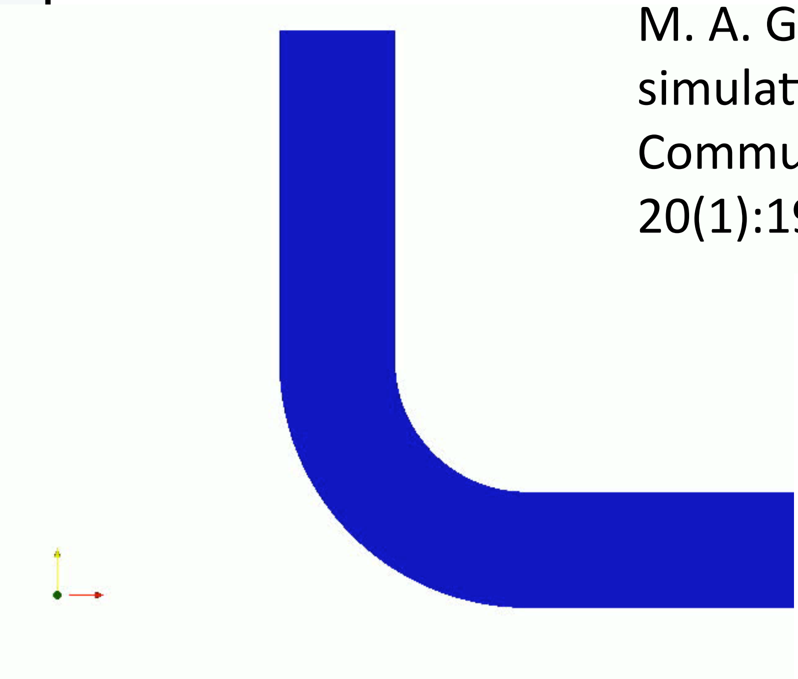
Isogeometric cohesive elements: 2D example



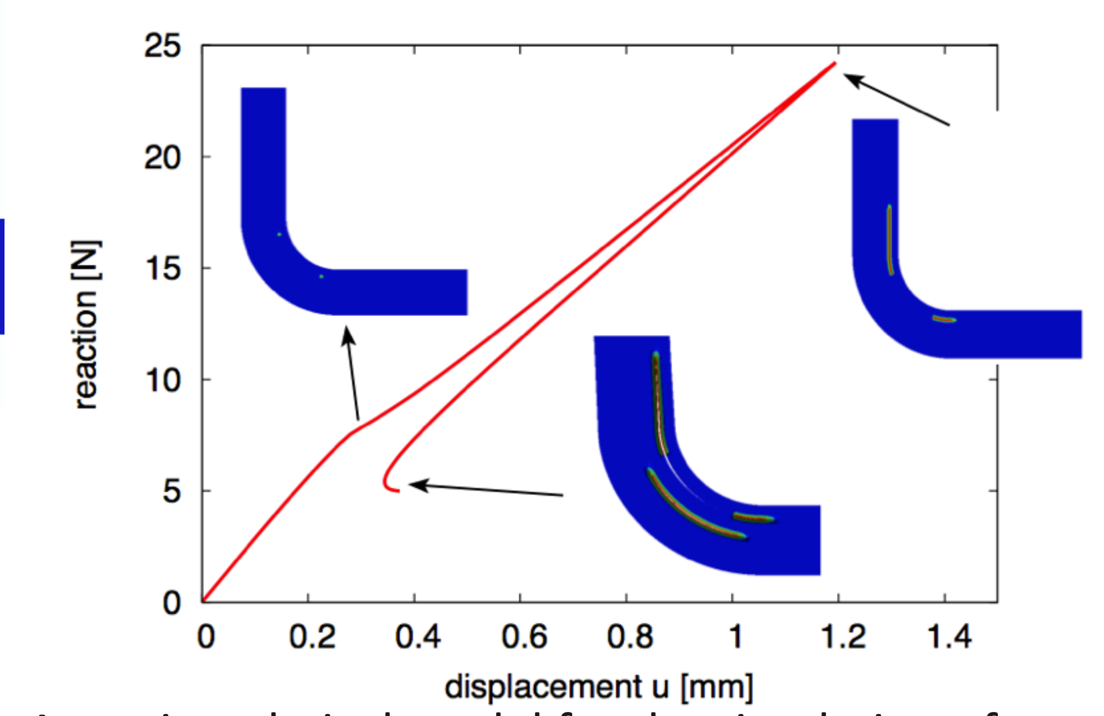


- exact geometry by NURBS
- It is straightforward to vary
  (1) number of plies and
  (2) # of interface elements:
- Suitable for parameter studies/design
- Cohesive law: bilinear law of Turon et al. 2006

Isogeometric cohesive elements: 2D example

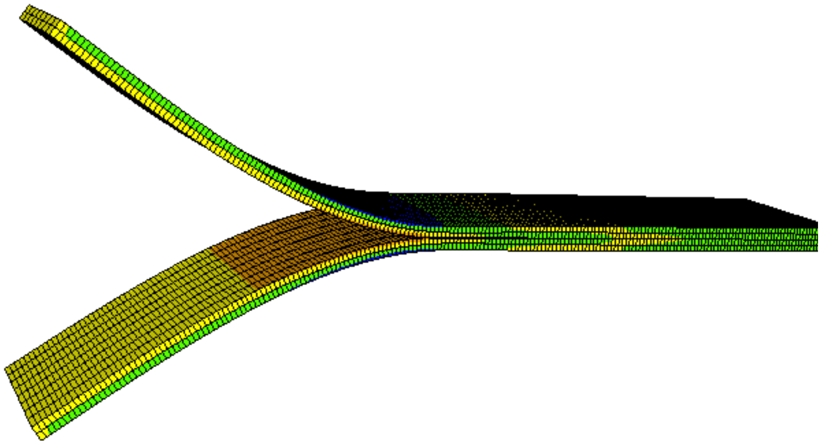


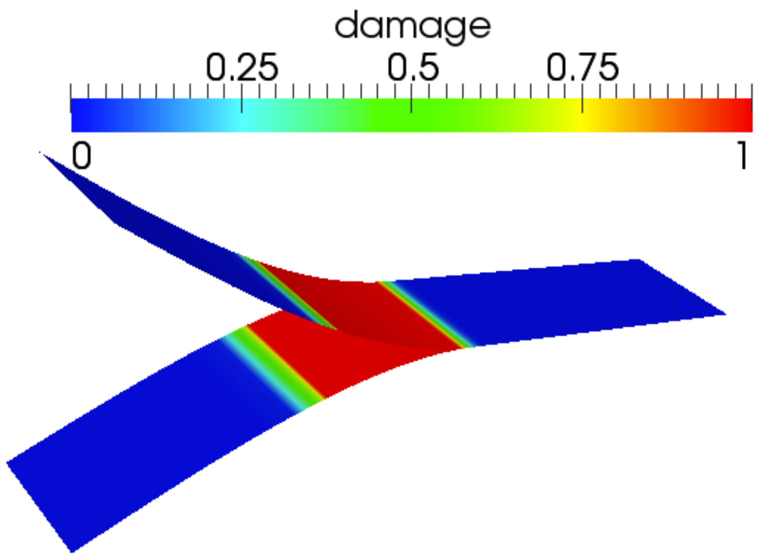
M. A. Gutierrez. Energy release control for numerical simulations of failure in quasi-brittle solids. Communications in Numerical Methods in Engineering, 20(1):19–29, 2004



G. Wimmer and H.E. Pettermann. A semi-analytical model for the simulation of delamination in laminated composites. Composites Science &Technology, 68(12):2332 – 2339, 2008.

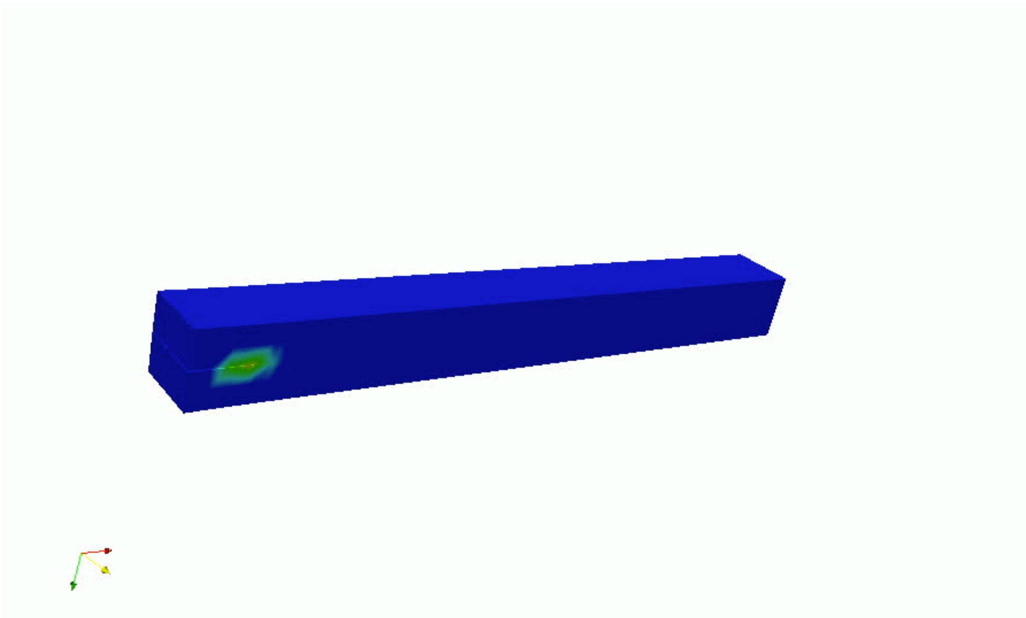
Isogeometric cohesive elements: 3D example with shells

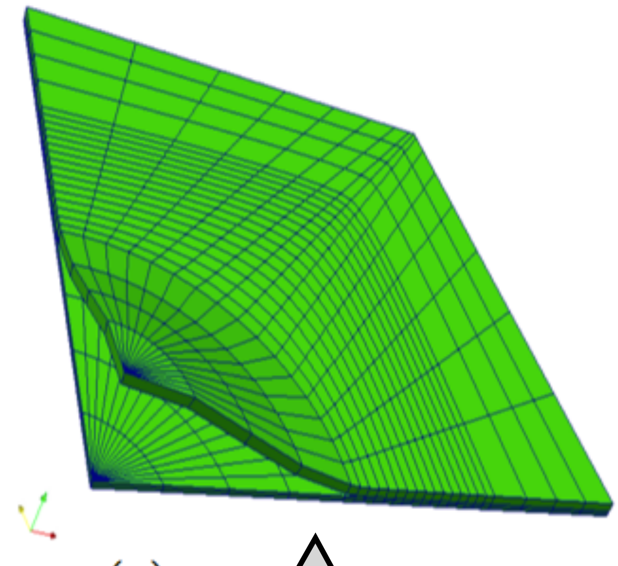




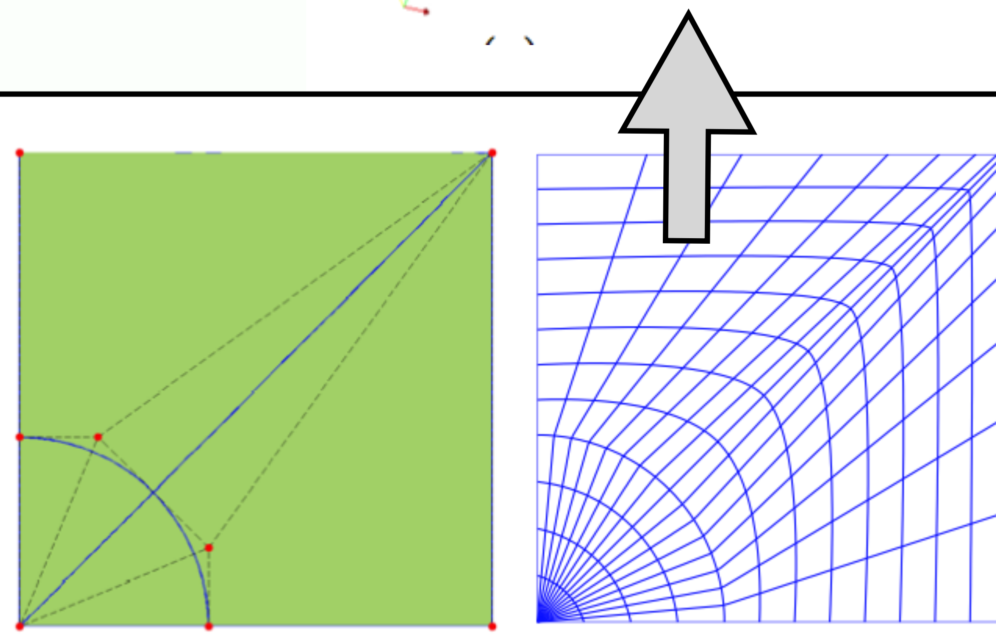
- Rotation free B-splines shell elements (Kiendl et al. CMAME)
- Two shells, one for each lamina
- Bivariate B-splines cohesive interface elements in between

Isogeometric cohesive elements: 3D examples

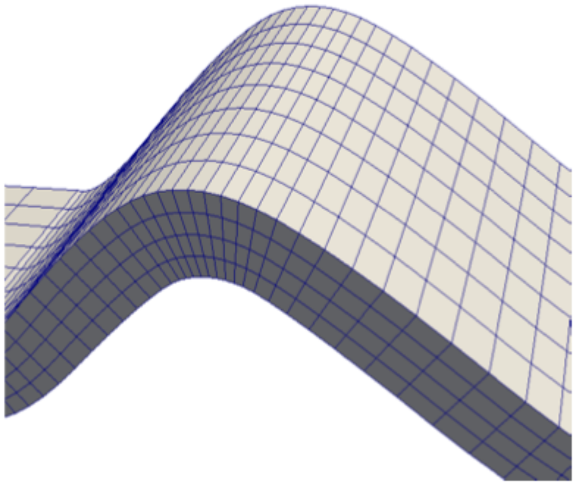




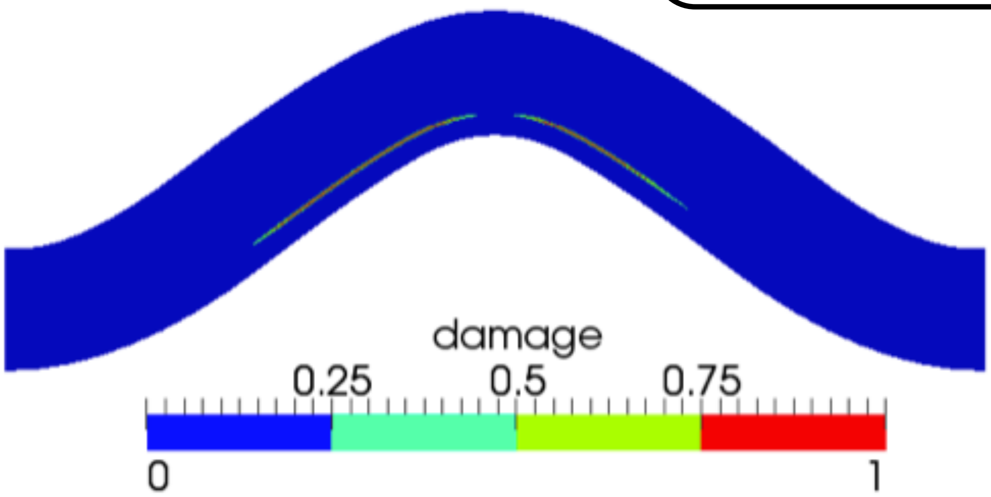
- cohesive elements for 3D meshes the same as 2D
- large deformations
- suitable: delamination buckling analysis



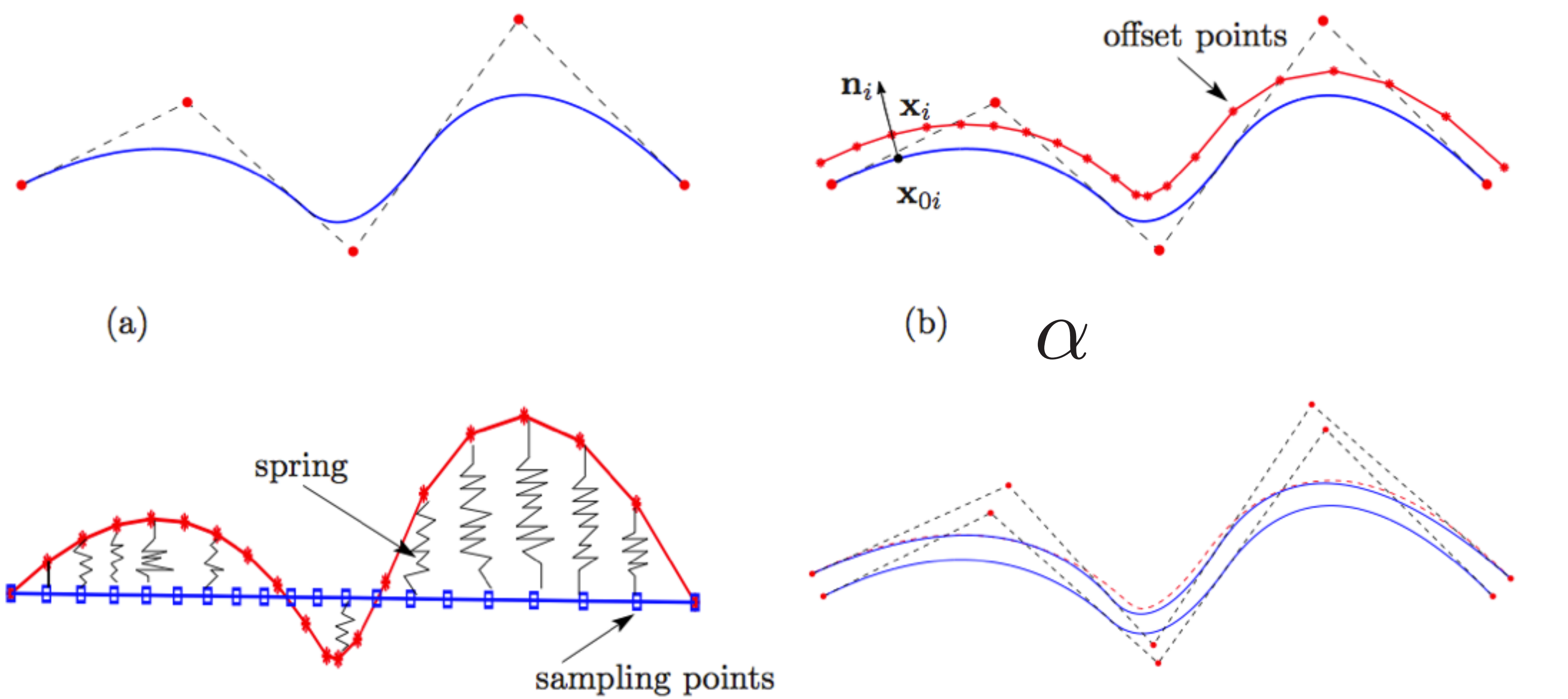
Isogeometric cohesive elements



- singly curved thick-wall laminates
- geometry/displacements: NURBS
- trivariate NURBS from NURBS surface
- cohesive surface interface elements
- compression test



Curve offseting

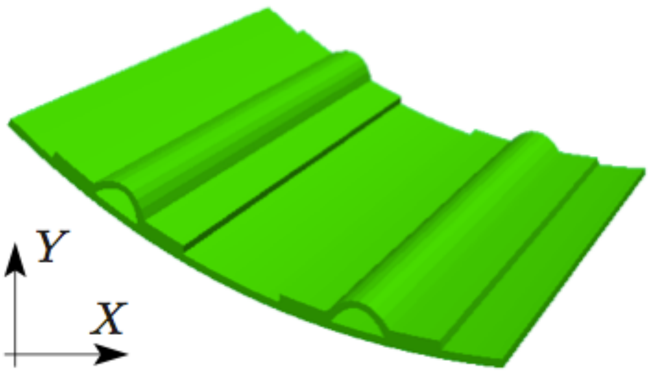


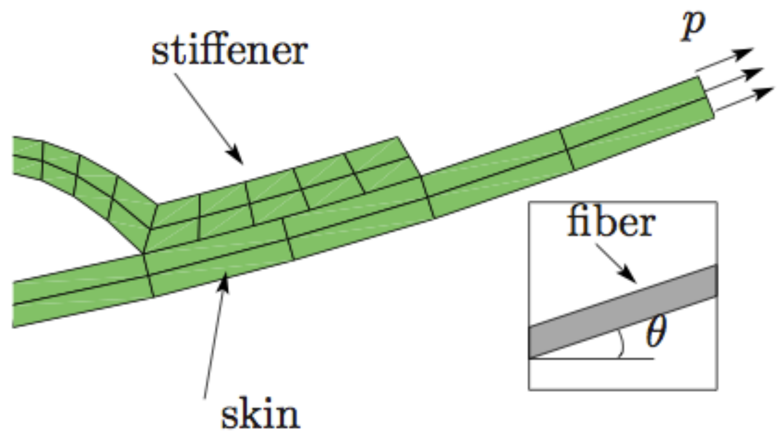
(a) (b) (c) (d)

α

(*)V. P. Nguyen, P. Kerfriden, S.P.A. Bordas, and T. Rabczuk. An integrated design-analysis framework for three dimensional composite panels. Computer Aided Design, 2013. submitted.

Curve offseting





$$E(\mathbf{B}) = \frac{1}{2}\sum_{i}^{m} k_s u_i(\mathbf{B})^2$$ -> min

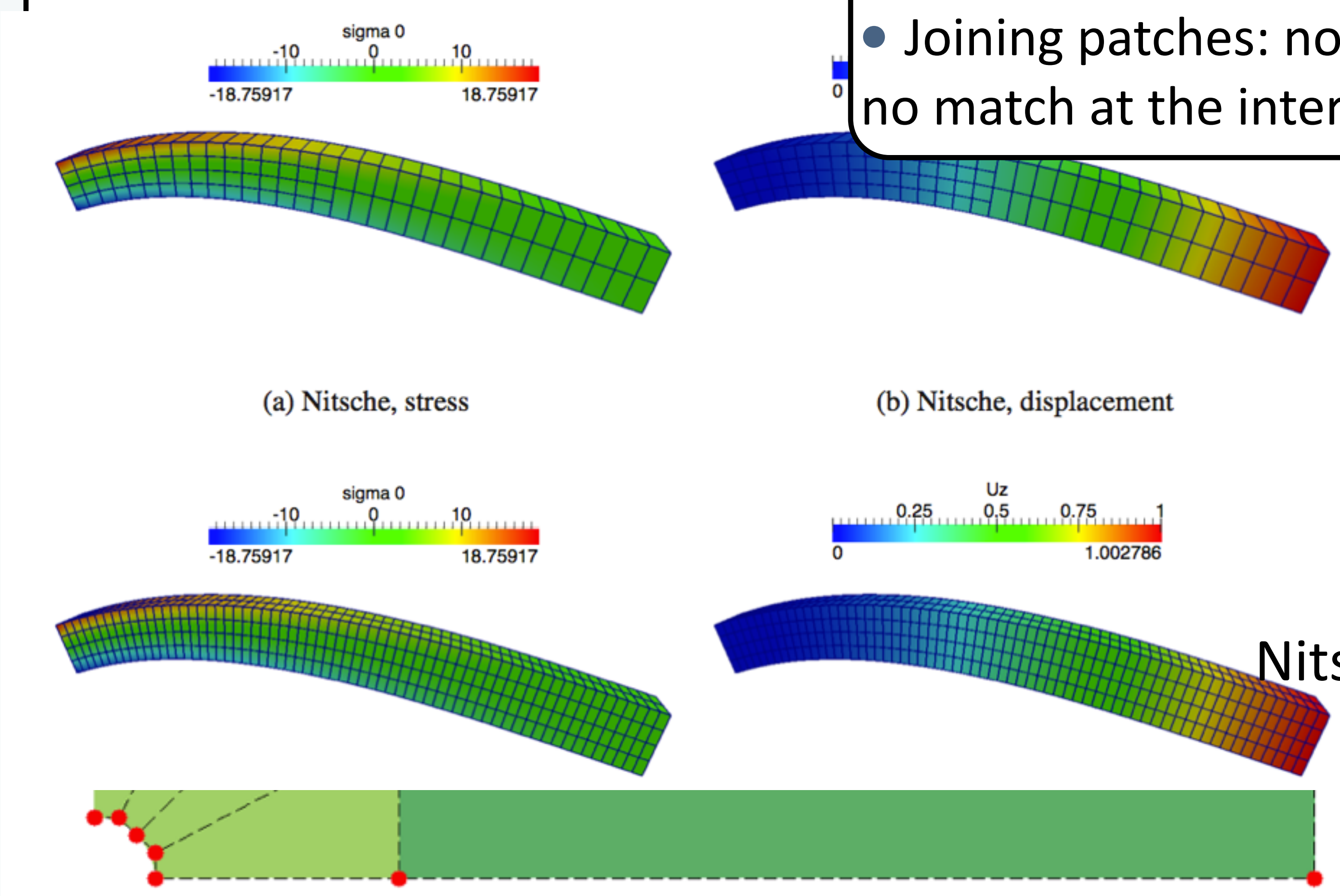
$$\mathbf{B}^{(1)} = \mathbf{B}^{(0)} - \gamma^{(0)}\nabla E^{(0)}$$

$$\gamma^{(0)} = \arg\min_{\gamma^{(0)}} E(\mathbf{B}^{(0)} - \gamma^{(0)}\nabla E^{(0)})$$

gradient decent method with line search

Multi patch NURBS

- Tensor-product: 4-sided shape
- Complex geom: multi-patch
- Each patch: its own parametrisation
- Joining patches: not trivial if there is no match at the interface



(a) Nitsche, stress

(b) Nitsche, displacement

Nitsche's method

Concluding remarks

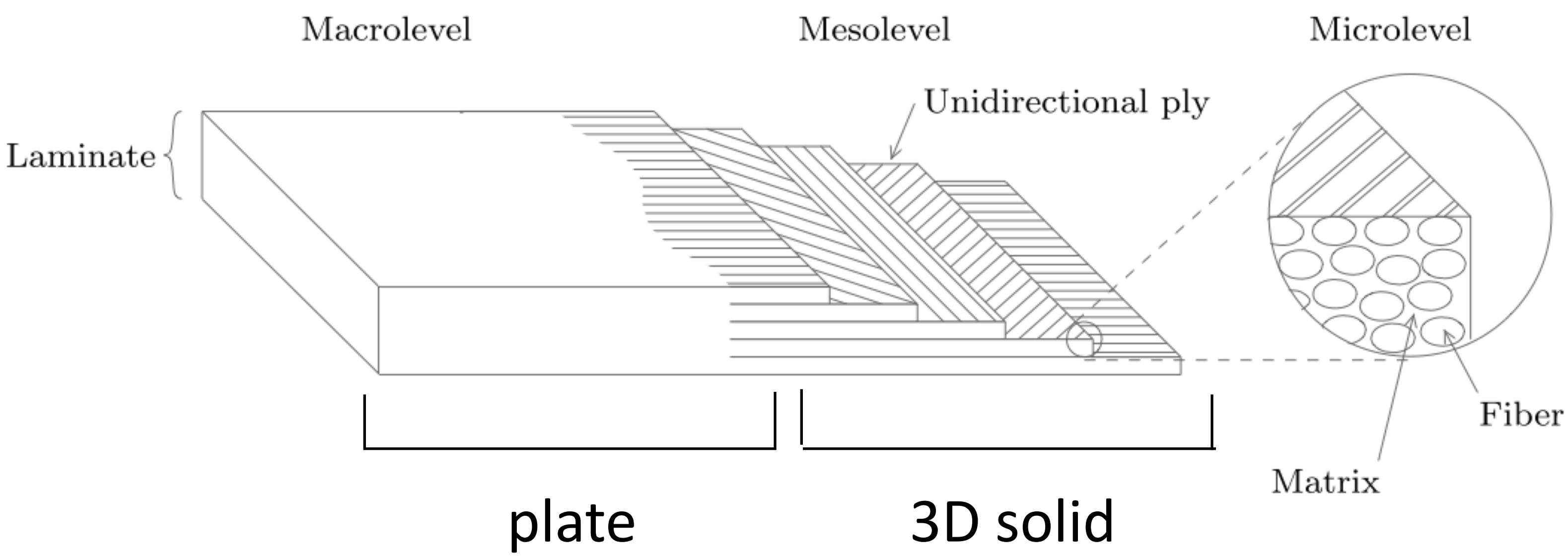
For composite laminates modeling, NURBS IGA offers

- fast pre-processing: automatic generation of interface el.
- link to CAD data: ideal for design-analysis cycles
- high order NURBS elements: highly accurate derivative fields
- less expensive than low order Lagrange elements
- geometry: exactly represented

- Tensor-product: no local refinement

- T-splines: complex algorithms
- Hierarchical B-splines
- Discontinuous Galerkin methods (NURBS)

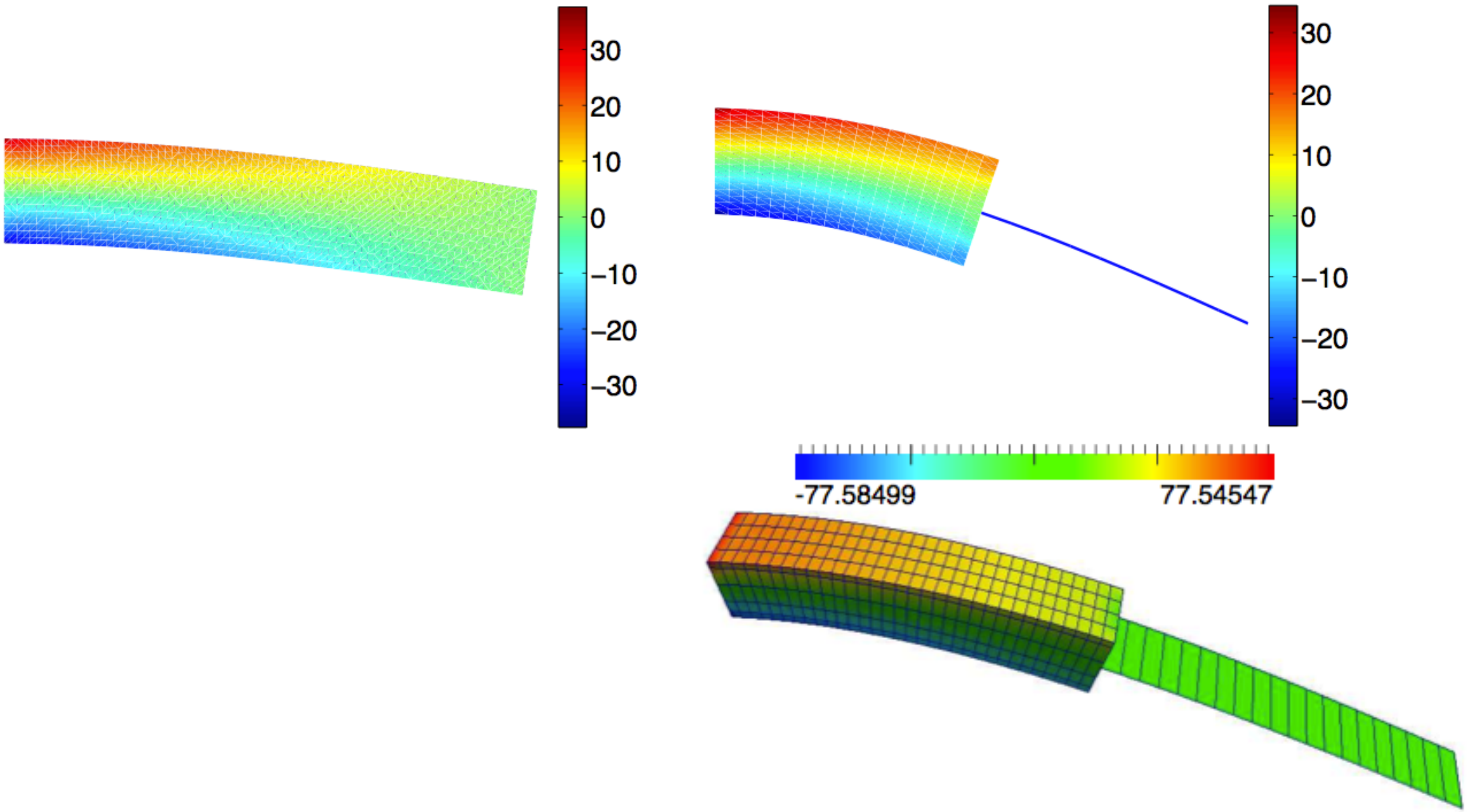
On-going and future work



- Multi model coupling: plate (macrolevel) and refined 3D continuum models (mesolevel)
- Macrolevel: through-thickness homogenisation can be used
- Conforming coupling: coupling via an interface
- Non-conforming coupling: 3D model placed anywhere on a

Multi model coupling with Nitsche's method

$$\int_{\Omega} \delta \boldsymbol{\epsilon}^{\mathrm{T}} \boldsymbol{\sigma} \mathrm{d}\Omega - \int_{\Gamma_*} [\![\delta \mathbf{u}]\!]^{\mathrm{T}} \mathbf{n} \{\boldsymbol{\sigma}\} \mathrm{d}\Gamma - \int_{\Gamma_*} \{\delta \boldsymbol{\sigma}\}^{\mathrm{T}} \mathbf{n}^{\mathrm{T}} [\![\mathbf{u}]\!] \mathrm{d}\Gamma + \int_{\Gamma_*} \alpha [\![\delta \mathbf{u}]\!]^{\mathrm{T}} [\![\mathbf{u}]\!] \mathrm{d}\Gamma = \int_{\Gamma_t} \delta \mathbf{u}^{\mathrm{T}} \bar{\mathbf{t}} \mathrm{d}\Gamma$$




Thank You!