Phase field modelling of fracture in solids

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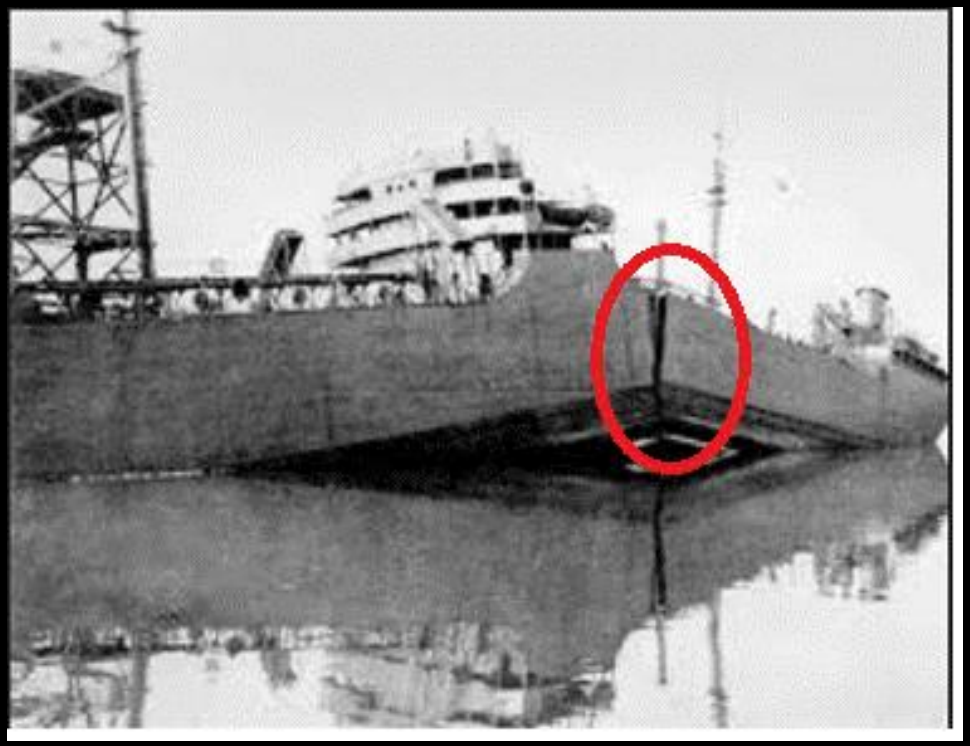
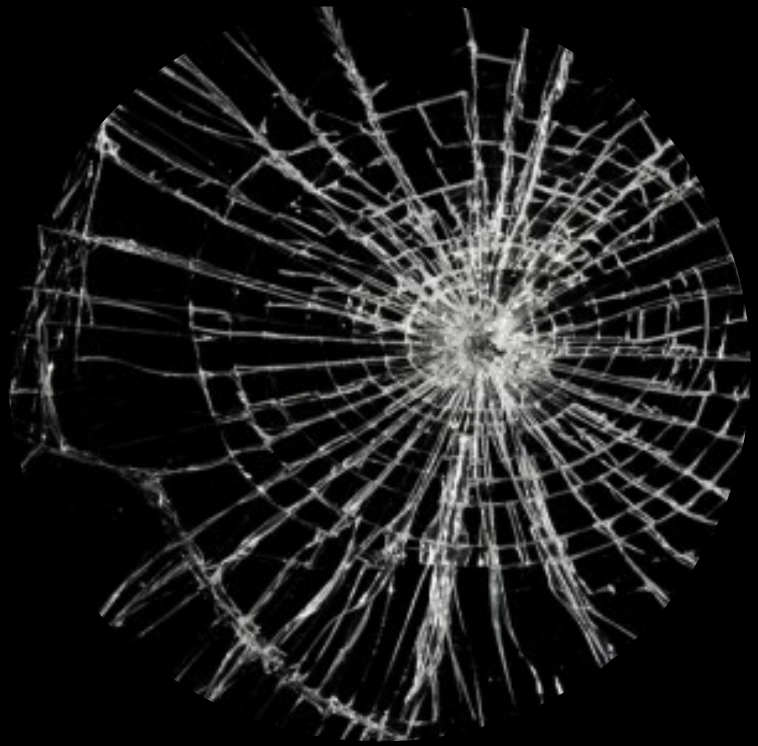
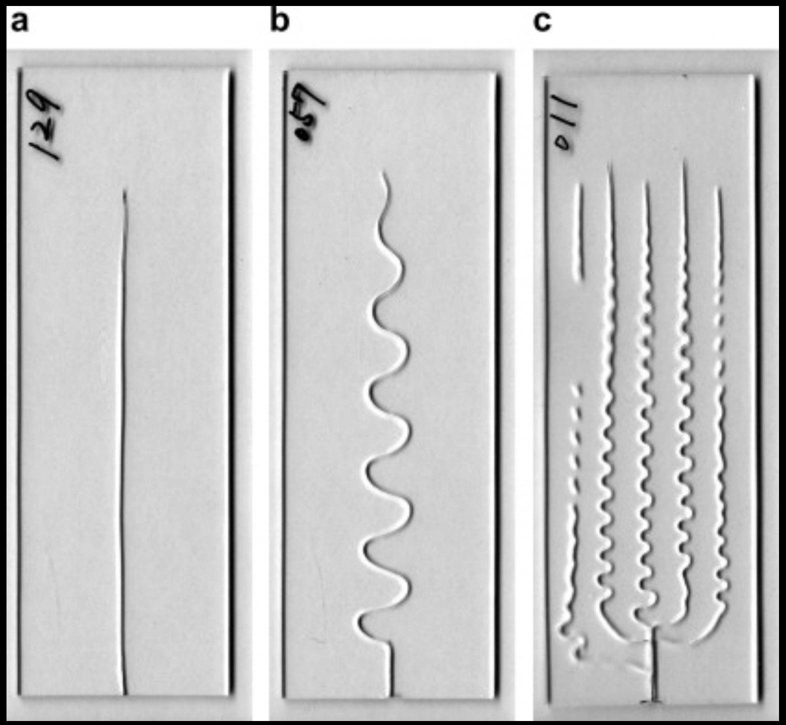
Australian Government
Australian Research Council

Fracture of solids: an important problem







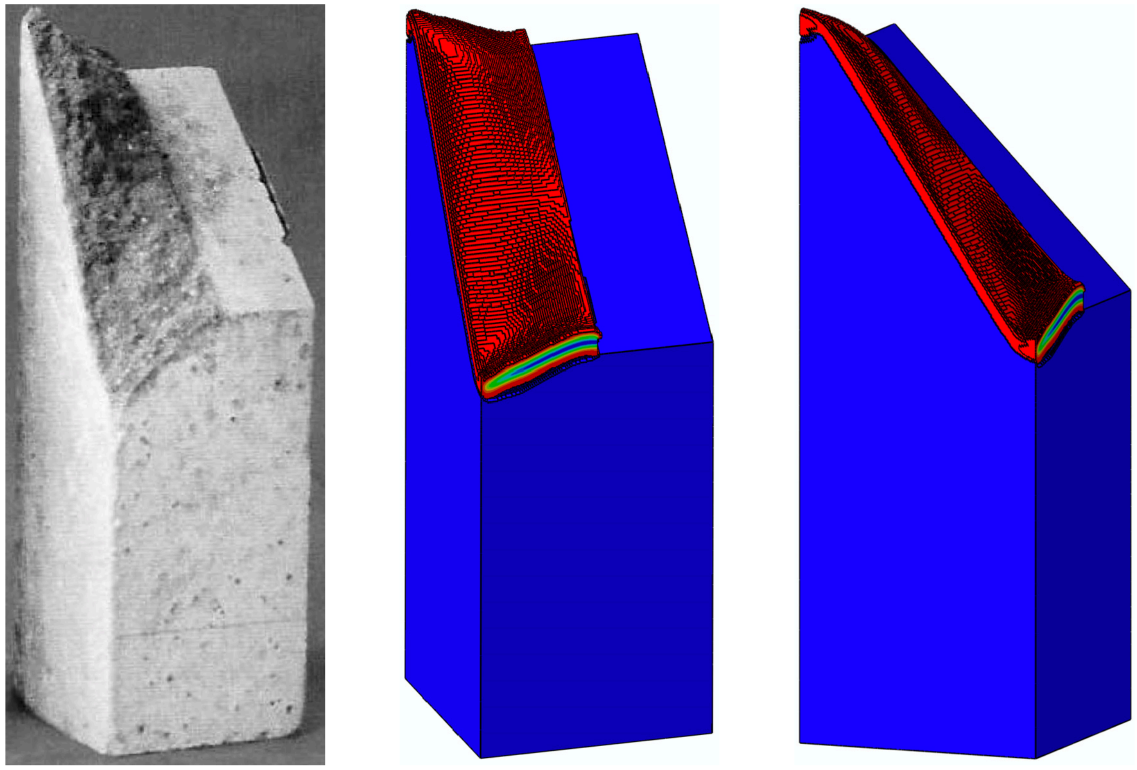
loss of integrity

catastrophic failure of structures/materials

initiation/propagation of cracks: **fascinating/challenging topic**

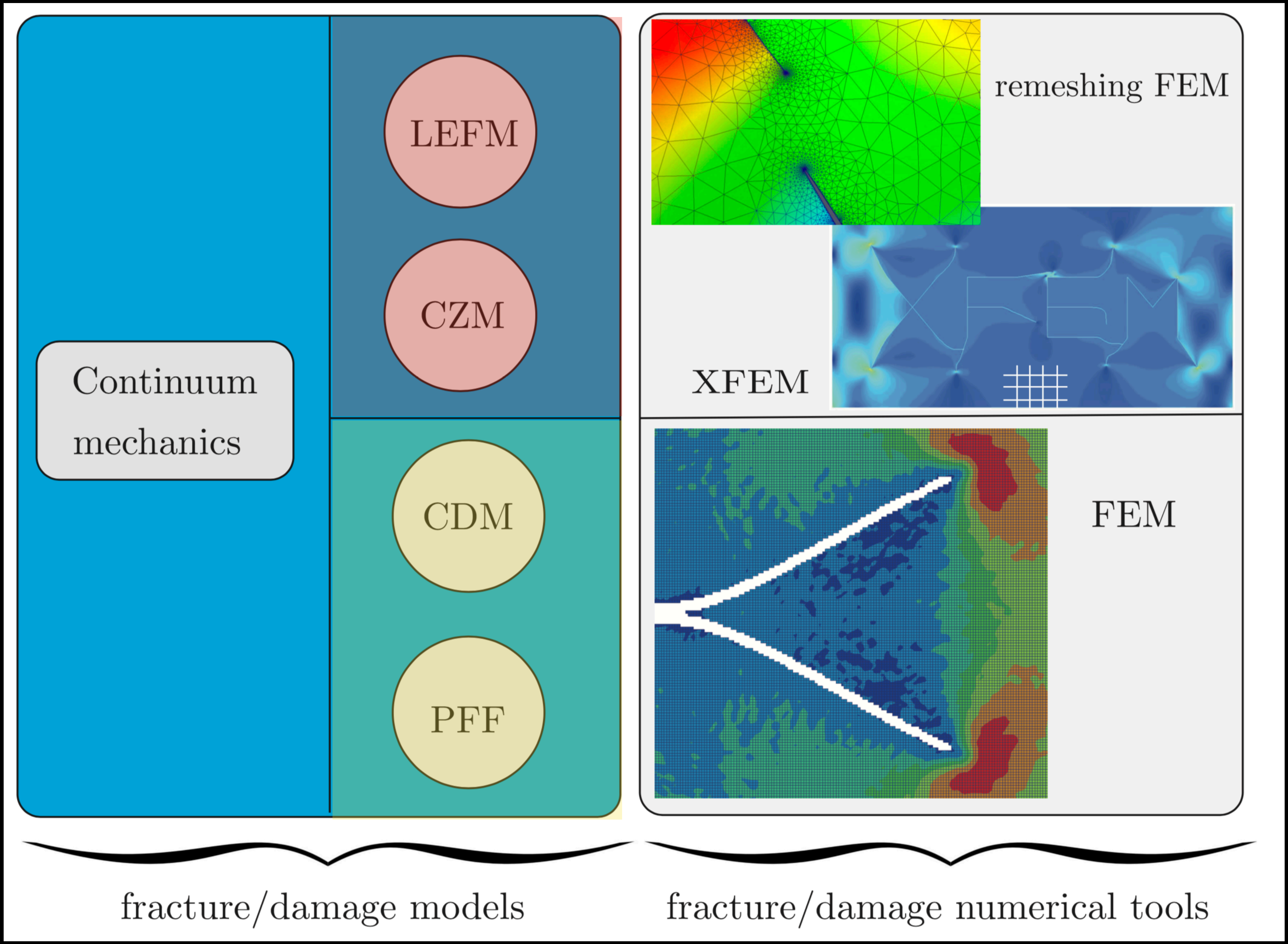
Aims: a good fracture model

a **physically sound model for fracture**,
a **robust** and **objective numerical method** to analyse crack problems
generality (2D, 3D, ductile/brittle fracture etc.**)**
handles both **crack initiation/propagation**
least amount of parameters

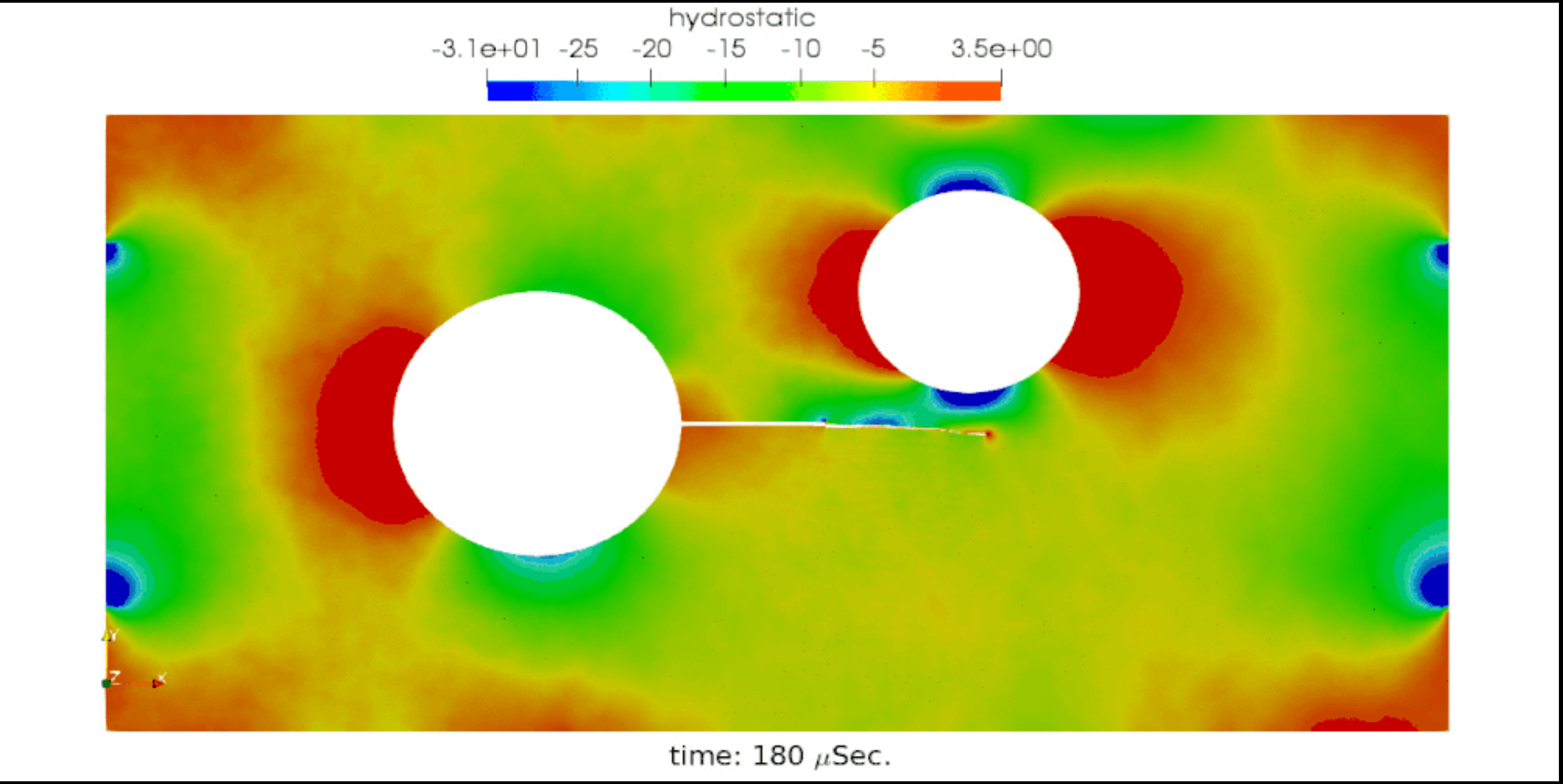


(a) test (b) view 1 (c) view 2

Continuum fracture/damage models

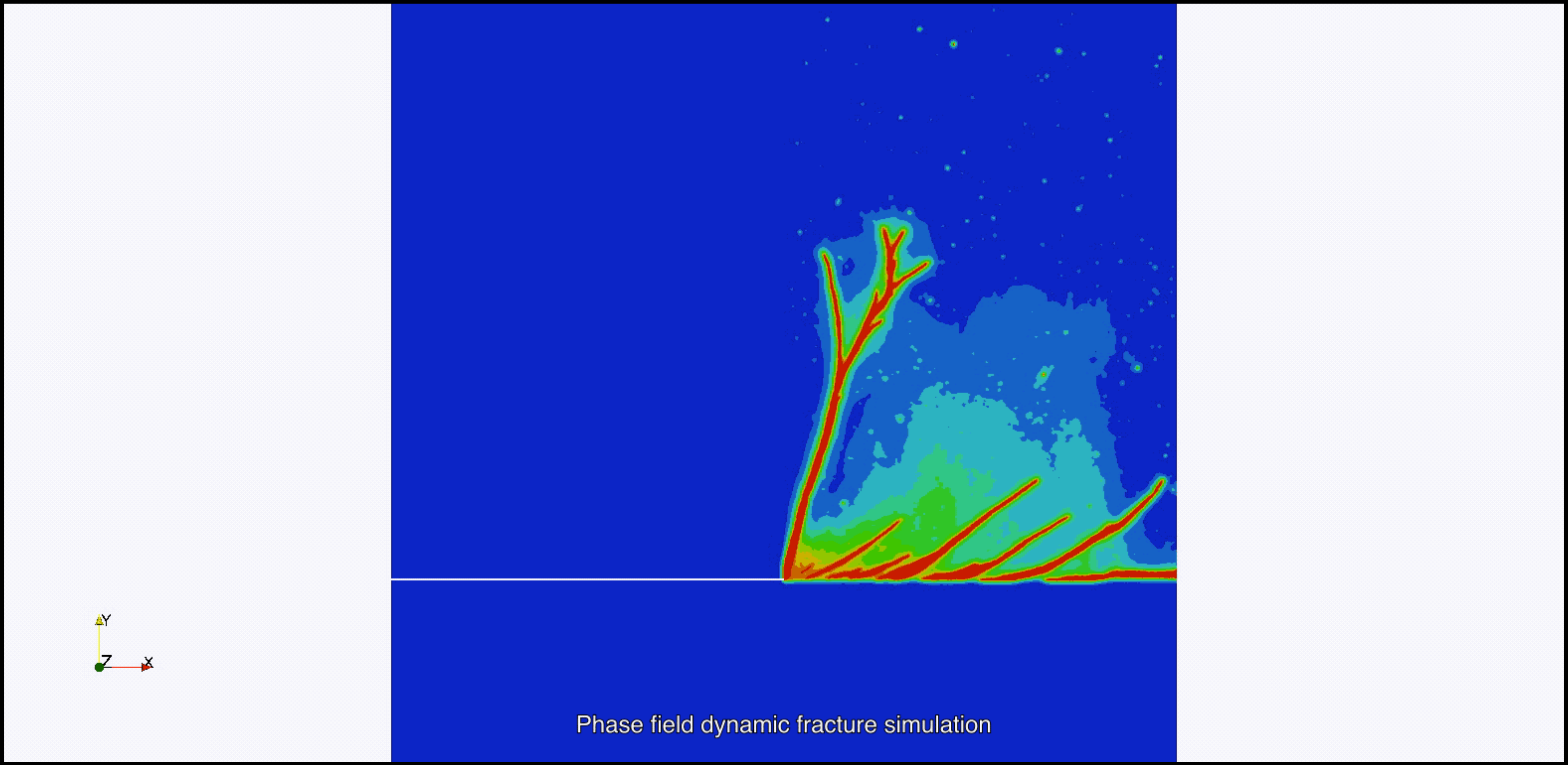


Phase field fracture: why?

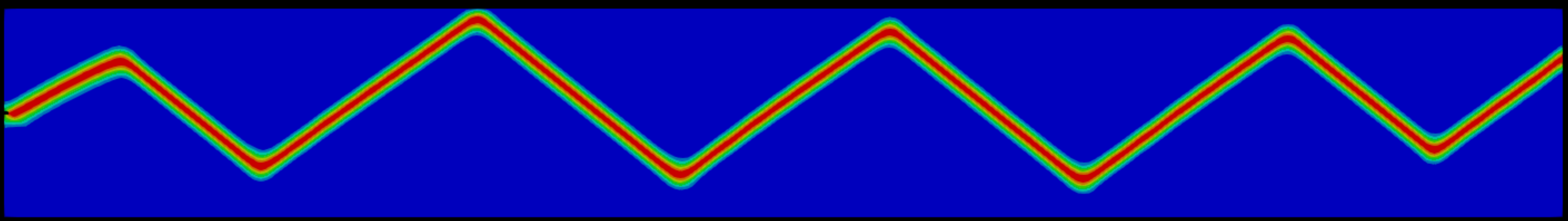


DYNAMIC CRACK ARREST

Phase field fracture: why?

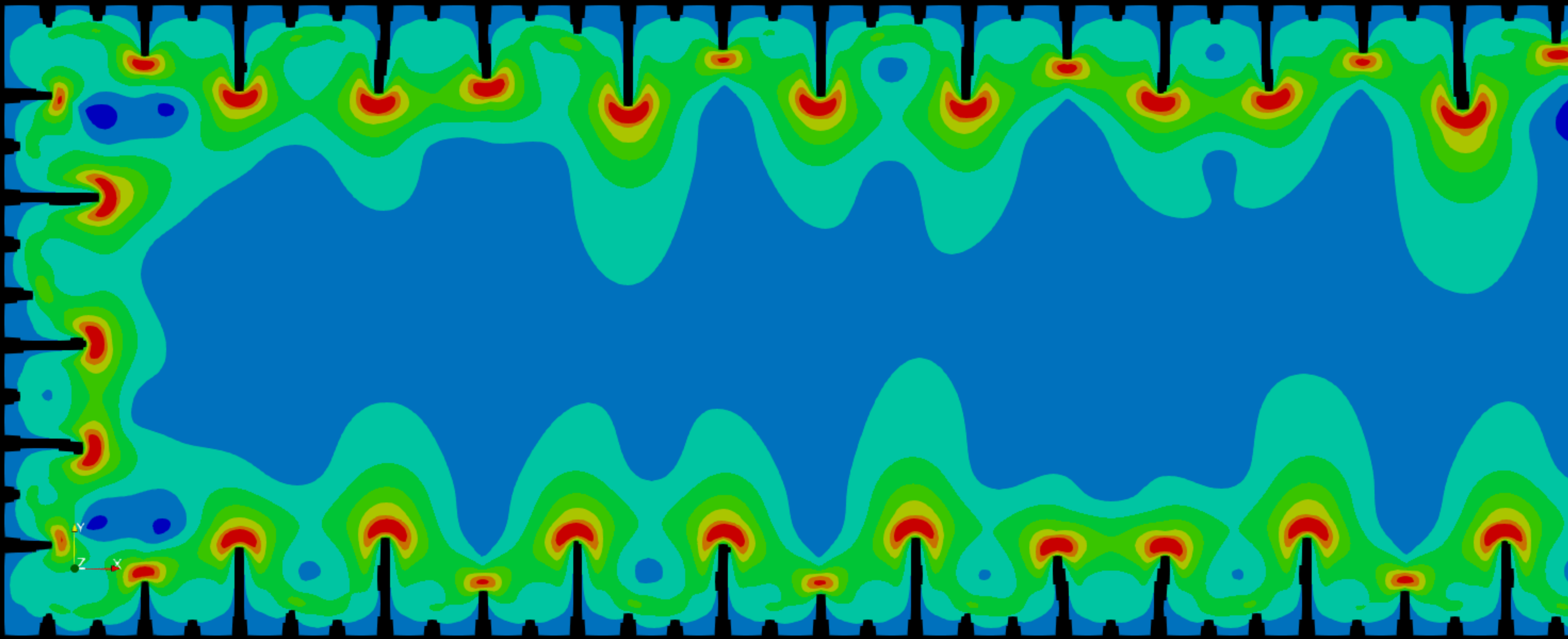


Phase field fracture: why?



CRACK KINKING

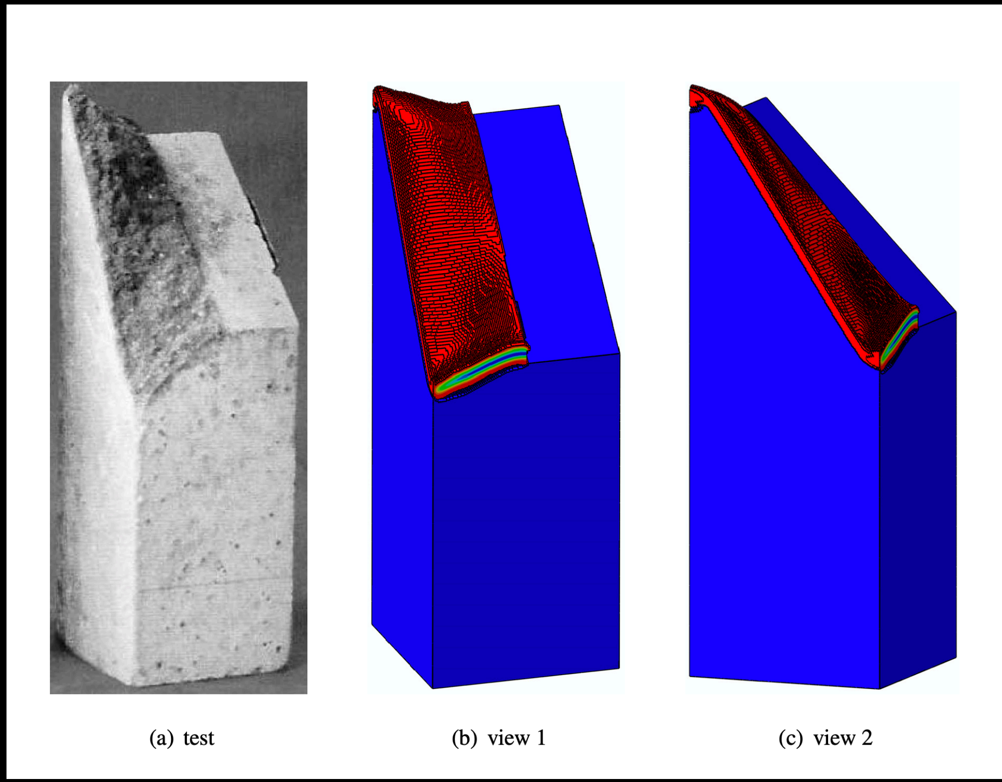
Phase field fracture: why?



THERMAL QUENCHING

Phase field fracture: why?

complex crack patterns without 'tricks'
simplicity and **generality** (2D, 3D & parallelisation)
mesh **convergent** results (crack profile, crack velocities)
rigorous mathematics/physics behind
robustness
can be easily coded in **commercial FE codes**



A brief history

AA Griffith
1893 – 1963

JJ Marigo

Ecole polytechnique

B Bourdin
Louisiana State Uni.

C Miehe
(1956-2016)

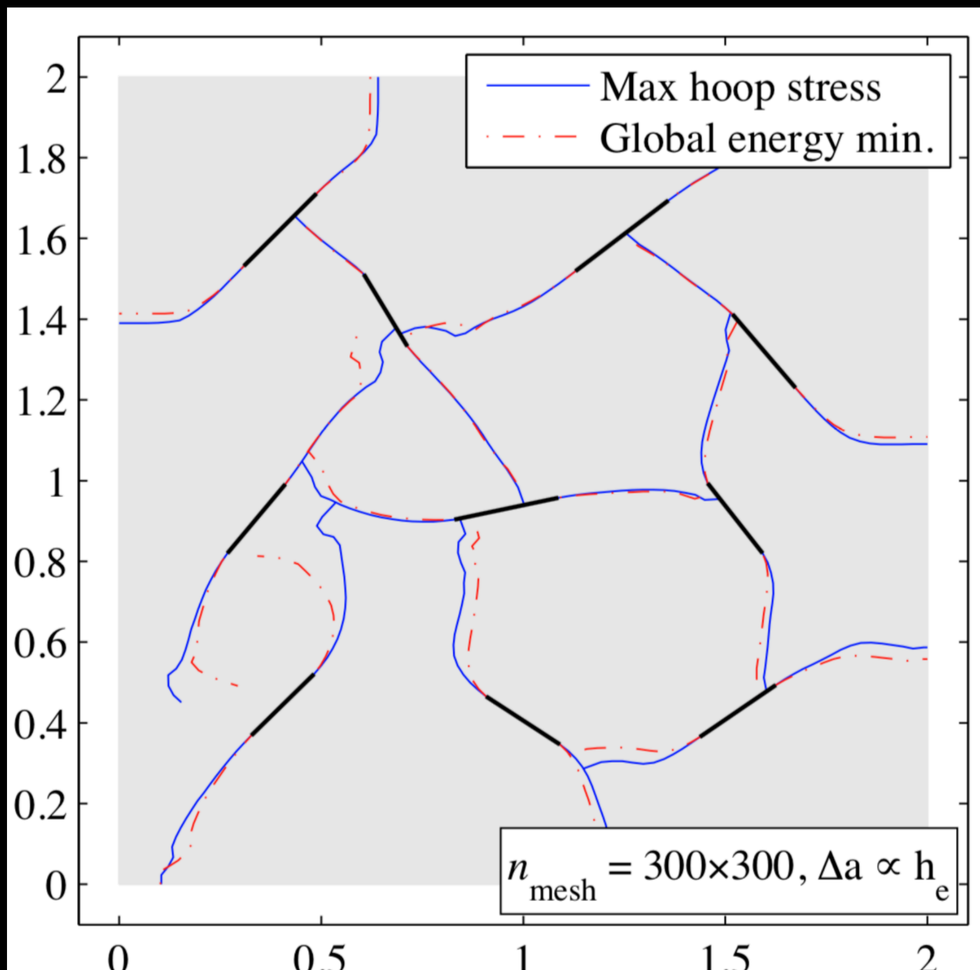
Energy method for brittle fracture (glass) 1922

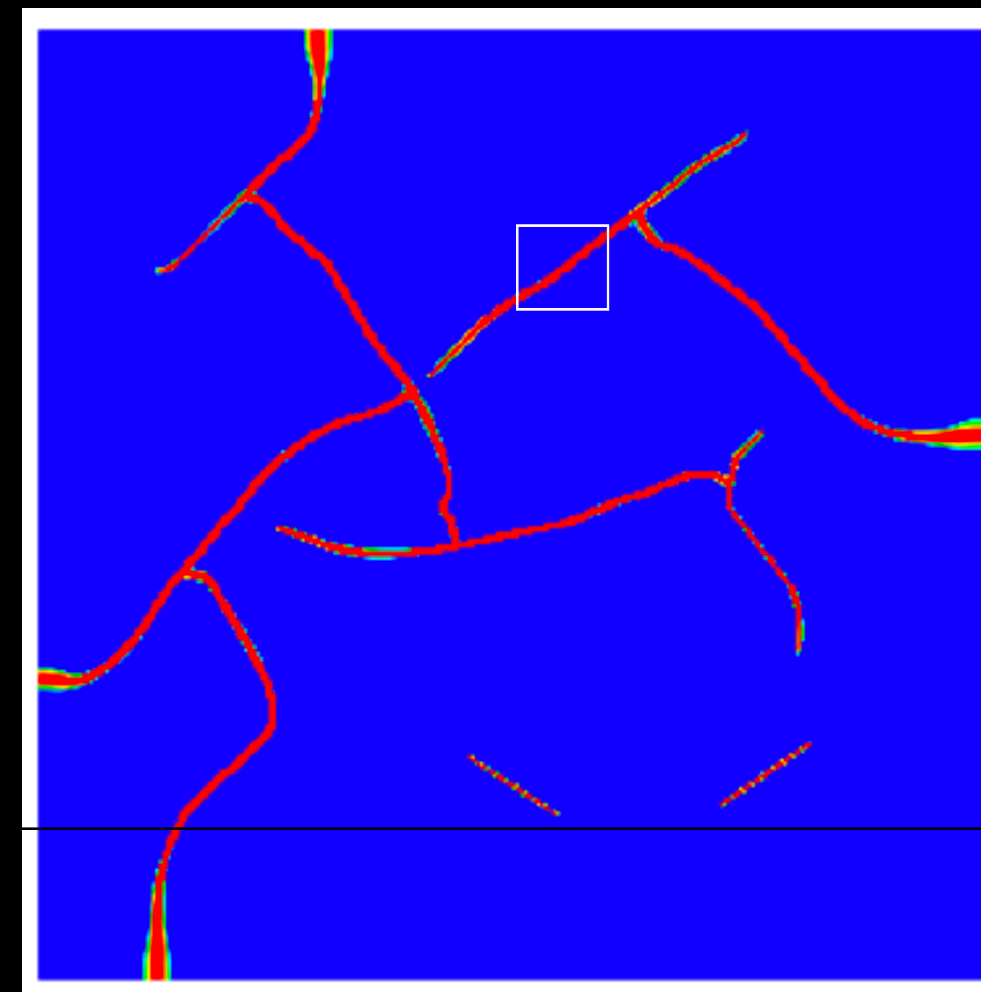
Phase-field implementation of Marigo's theory, 2000

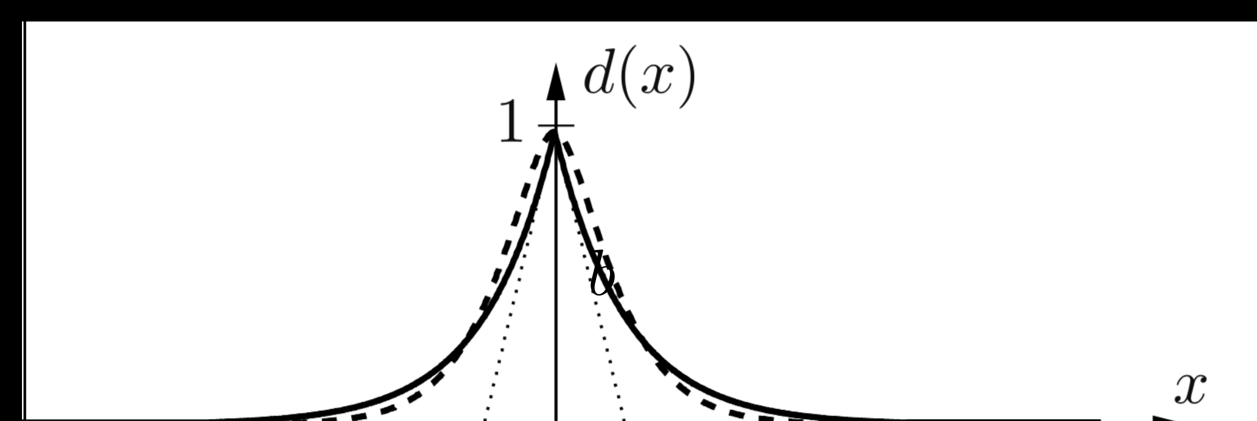
Variational formulation of Griffith's theory, 1998

Reformulation of phase-field fracture accessible to engineers 2010

PFM's key idea 1: crack geometry approximation

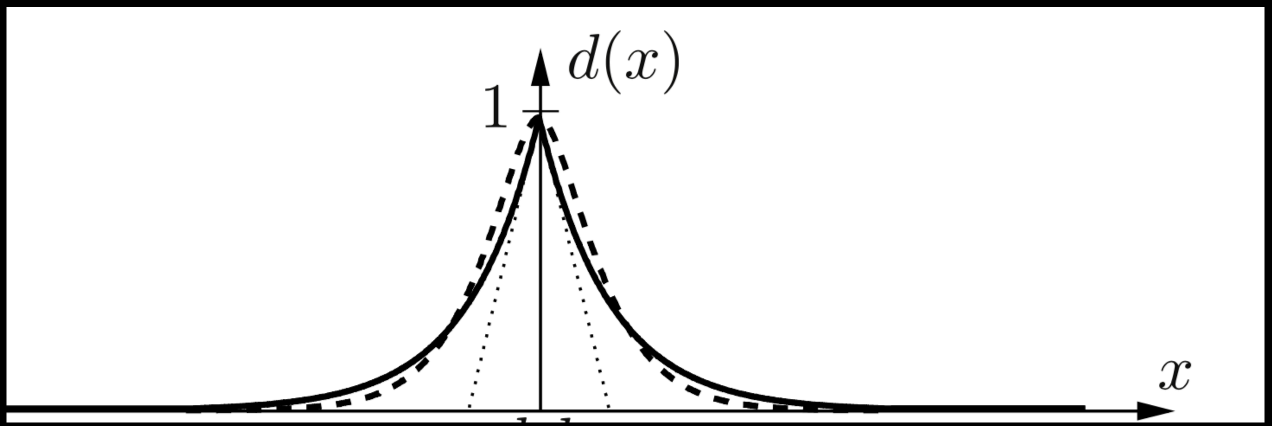






PFM's key idea 1: crack geometry approximation

$$d(x) = \exp\left(-\frac{|x|}{b}\right)$$



$$\frac{1}{b}d(x) - bd''(x) = 0$$

$$I[d(x)] = \int_{-\infty}^{+\infty} \frac{1}{2}\left[\frac{1}{b}d^2 + b[d'(x)]^2\right] dx$$

$$I[d(\boldsymbol{x})] = \int \frac{1}{2}\left[\frac{1}{b}d^2 + b\nabla d \cdot \nabla d\right] dV$$

PFM's key idea 2: Griffith's energy principle

Solutions are **minimiser** of the energy functional

$$\Psi(\boldsymbol{u}, \Gamma) = \int_{\Omega/\Gamma} \psi_0(\boldsymbol{\epsilon}(\boldsymbol{u}))\mathrm{d}V + \int_{\Gamma} G_c \mathrm{d}A - \mathcal{P}$$

strain energy

surface/fracture

GA Francfort, J-J Marigo. Revisiting brittle fracture as an energy minimization problem. *JMPS*, 1998.

Variational fracture with phase-field approximation of the crack surfaces

$$\int_{\Gamma} G_c \mathrm{d}A \approx \int_{\Omega} G_c \gamma(d; \nabla d) \mathrm{d}A$$

$$\gamma(d; \nabla d) = \frac{1}{c_\alpha}\left[\frac{1}{b}\alpha(d) + b\left|\nabla d\right|^2\right]$$

$$\Psi(\boldsymbol{u}, d) = \int_{\Omega} g(d)\psi_0(\boldsymbol{\epsilon}(\boldsymbol{u}))\mathrm{d}V + \int_{\Omega} G_c \gamma(d; \nabla d)\mathrm{d}V - \mathcal{P}$$

strain energy

surface/fracture energy

GA Francfort, J-J Marigo. Revisiting brittle fracture as an energy minimization problem. *JMPS*, 1998.

B. Bourdin, GA Francfort, and J-J Marigo. Numerical experiments in revisited brittle fracture. *JMPS*, 2000.

Governing equations

Governing equations

$$\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{b} = \boldsymbol{0} \qquad \text{in } \Omega$$

$$g'(d)\psi_0(\boldsymbol{\epsilon}) + G_c\, \delta_d \gamma = 0 \qquad \text{in } \mathcal{B}$$

$$\boldsymbol{\sigma} \cdot \boldsymbol{n} = \boldsymbol{t} \qquad \text{on } \partial\Omega_t$$

$$\frac{\partial \gamma}{\partial \nabla d} \cdot \boldsymbol{n}_{\mathcal{B}} = 0 \qquad \text{on } \partial\mathcal{B}$$

$$\boldsymbol{\sigma} = g(d)\frac{\partial \psi_0}{\partial \boldsymbol{\epsilon}}$$

$$\psi_0 = \frac{1}{2}\boldsymbol{\epsilon} : \mathbb{E}_0 : \boldsymbol{\epsilon}$$

$$\boldsymbol{\sigma} = g(d)\mathbb{E}_0 : \boldsymbol{\epsilon}$$

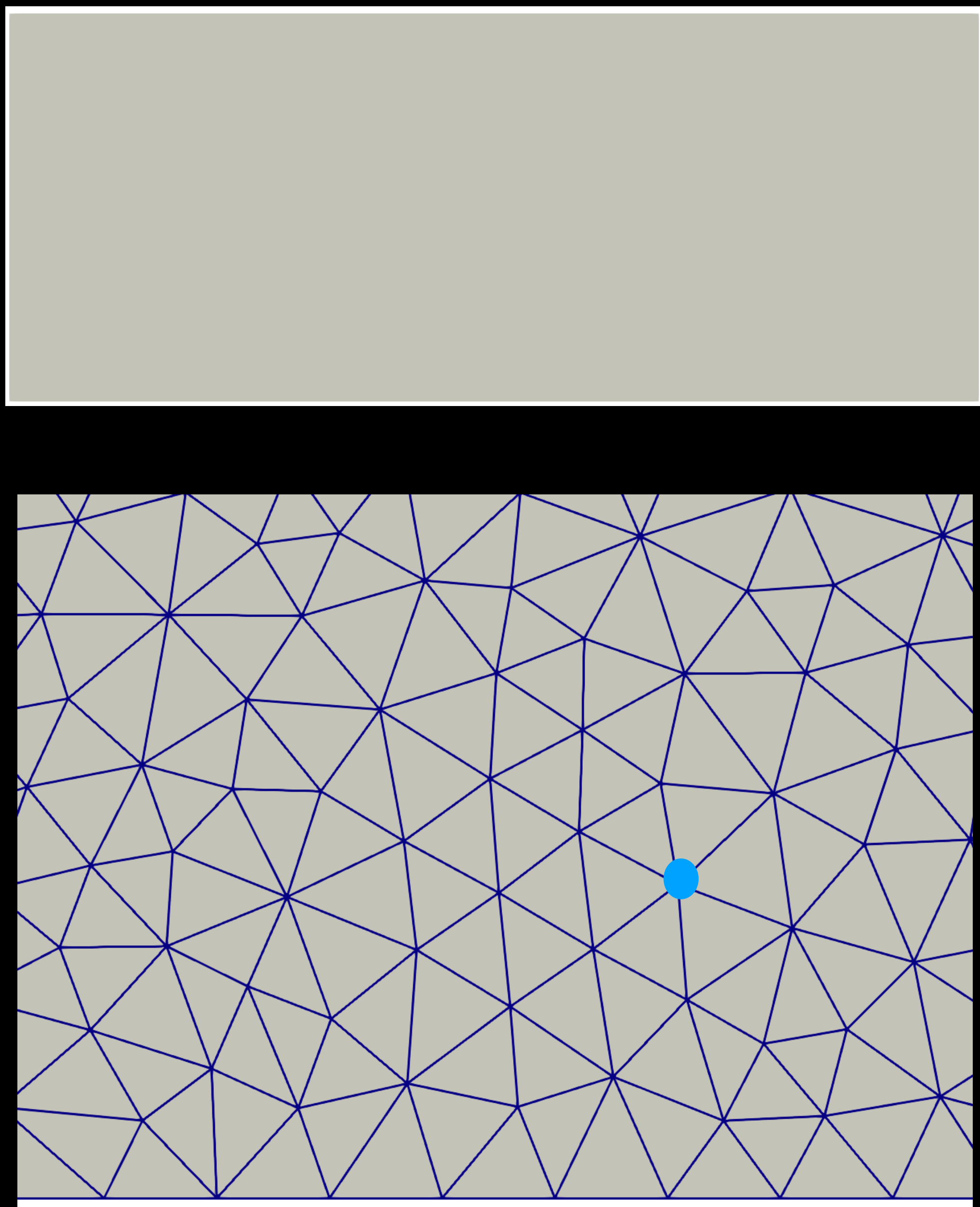
$$g(d) = (1-d)^2$$

$$\alpha(d) = d^2$$

$$\dot{d} \geq 0$$

only for brittle fracture

Implementation: multi-field elements & staggered solver



● (u_x, u_y, d)

Solve for displacements with damage fixed

$$\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{b} = \boldsymbol{0}$$

$$\boldsymbol{\sigma} = g(d^*)\mathbb{E}_0 : \boldsymbol{\epsilon}$$

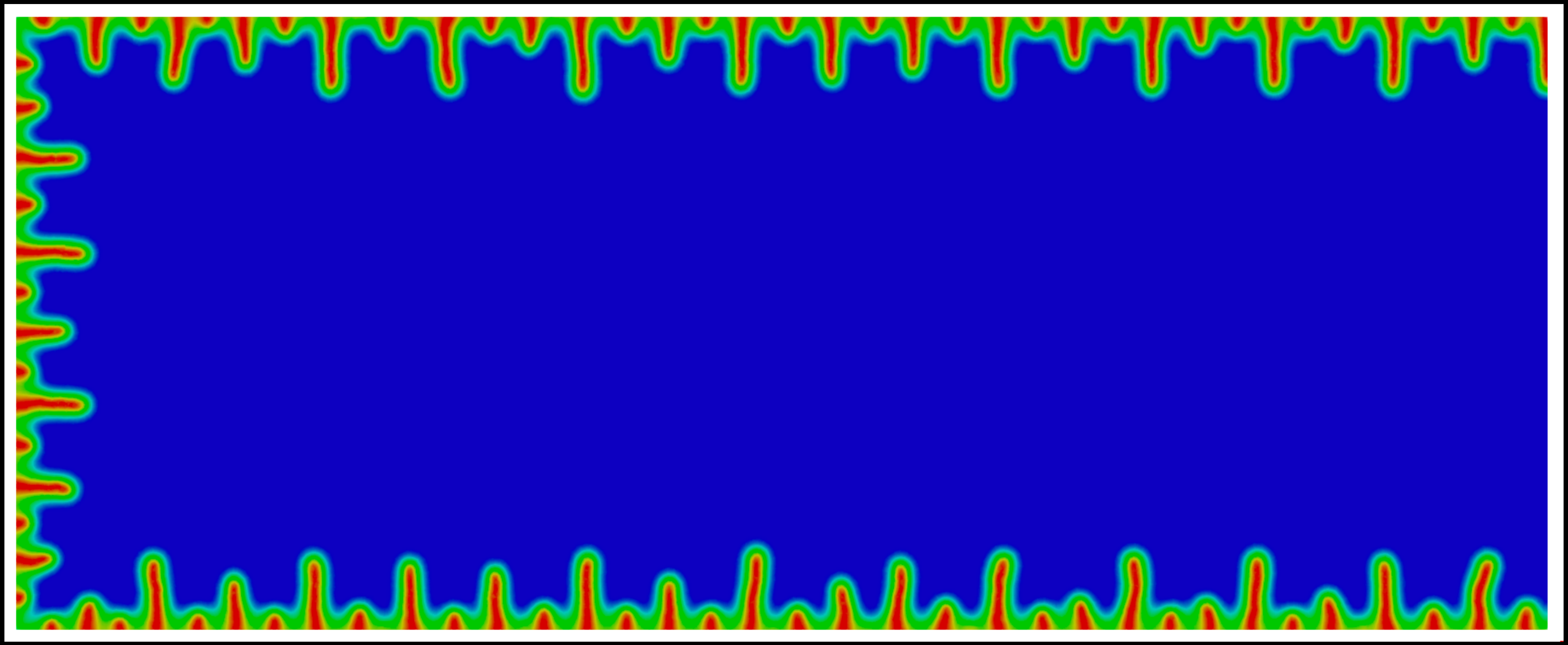
Solve for damage with new displacements

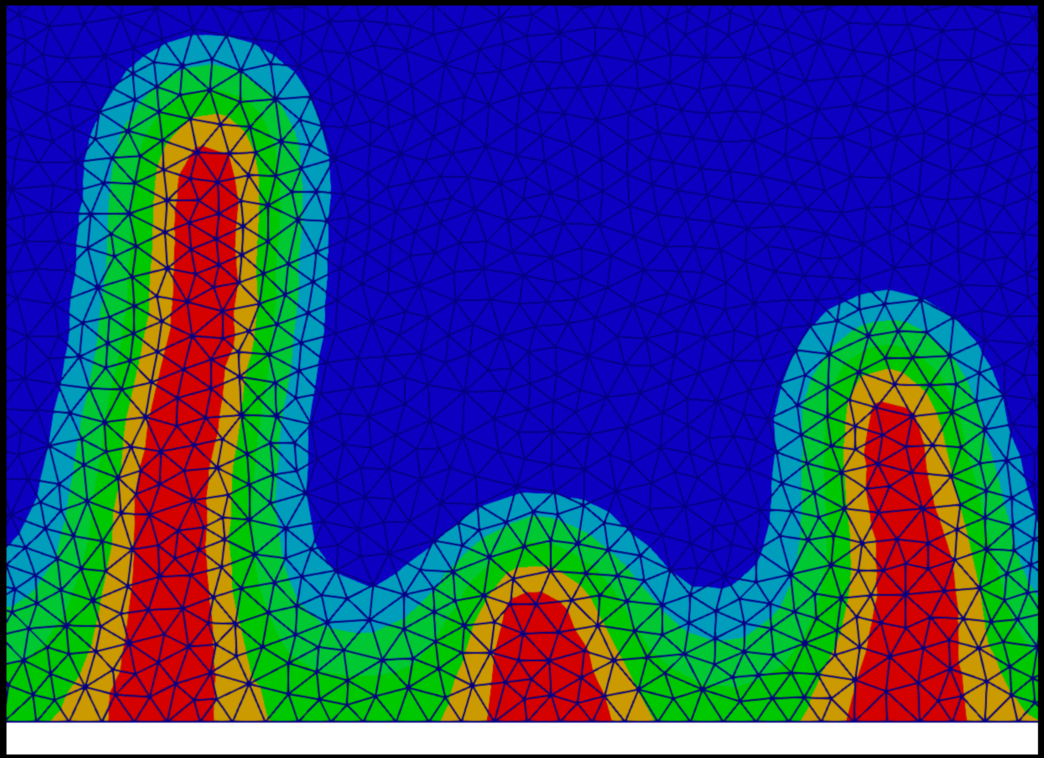
$$G_c \delta_d \gamma = -g'(d)\psi_0(\boldsymbol{\epsilon}(\boldsymbol{u}))$$

B. Bourdin, GA Francfort, and J-J Marigo. *JMPS*, 2000.

J.Y. Wu, V.P. Nguyen et al. Advances in Applied Mechanics, 2018

Cracks are region where damage is non-zero





many elements
across the damage band

How to select the value of the length scale?

	AT2	AT1
fracture type	brittle	brittle
Elastic domain	No	Yes
$\alpha(d)$	d^2	d
$\omega(d)$	$(1-d)^2$	$(1-d)^2$
c_α	2	8/3
length scale b	$b = \frac{27}{256} l_{\text{ch}}$	$b = \frac{3}{8} l_{\text{ch}}$
Damage bandwidth	∞	$4b$
Parameters	$E_0, \nu_0, \rho, G_{\text{f}}, b(f_t)$	$E_0, \nu_0, \rho, G_{\text{f}}, b(f_t)$

$$l_{\text{ch}} = \frac{E_0 G_f}{f_t^2}$$

Such length scales can be: too small

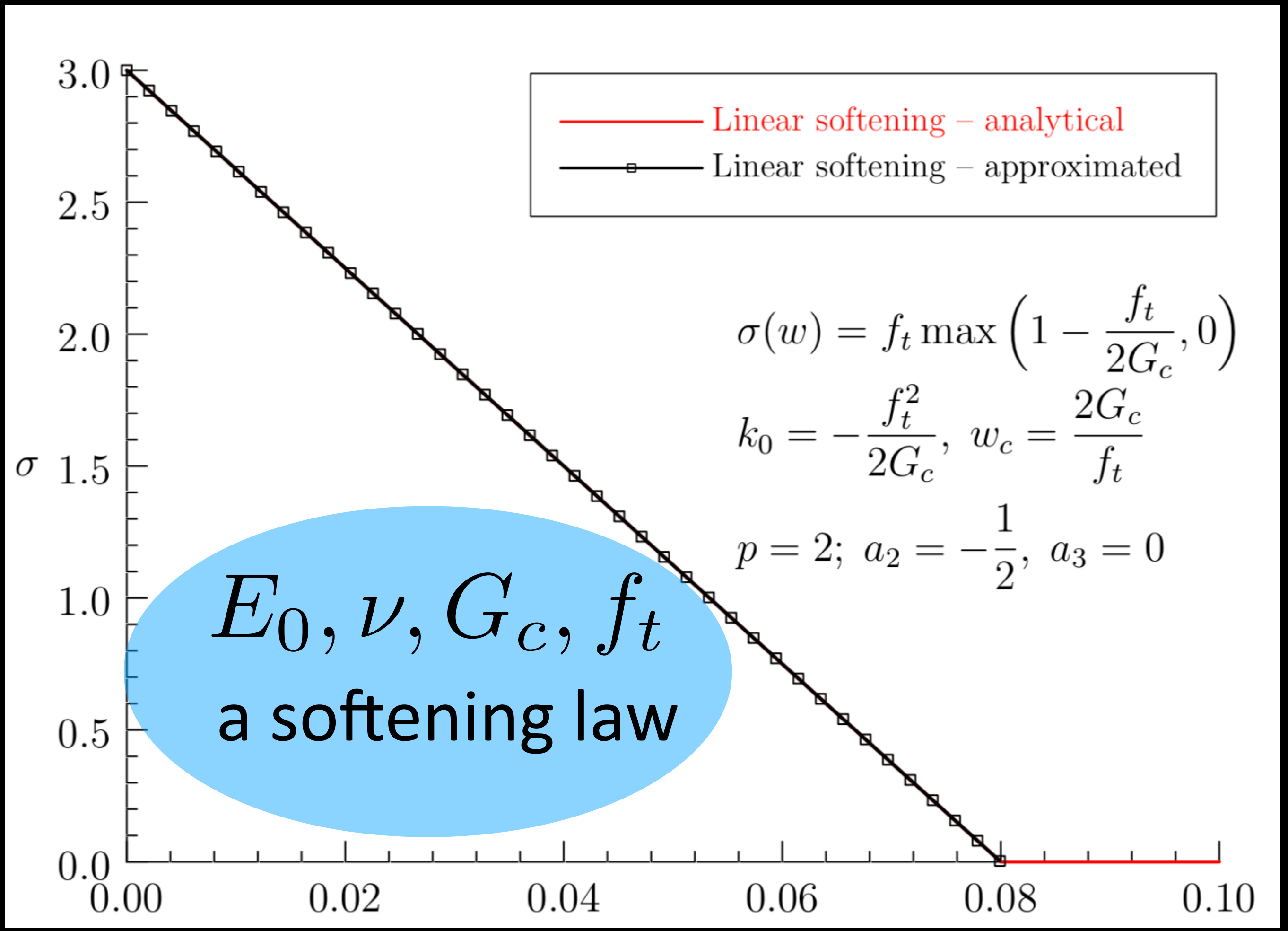
Such length scales can be: too big

PFM for quasi-brittle fracture: rational degradation function

$$g(d) = \frac{(1-d)^p}{(1-d)^p + a_1 d \cdot P(d)}$$

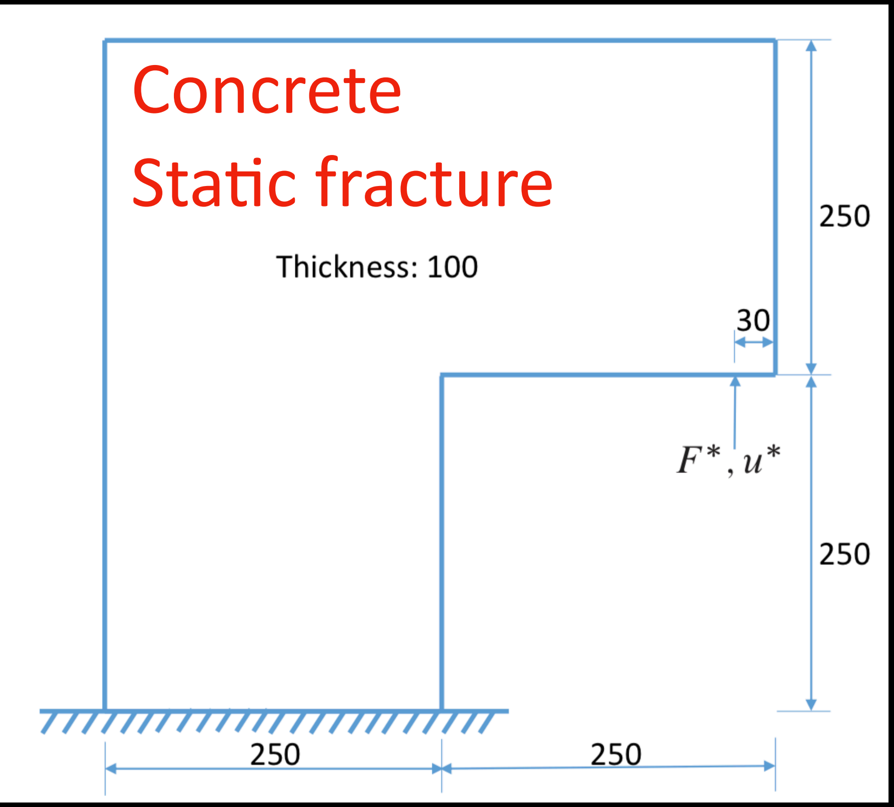
$$P(d) = 1 + a_2 d + a_3 d^2$$

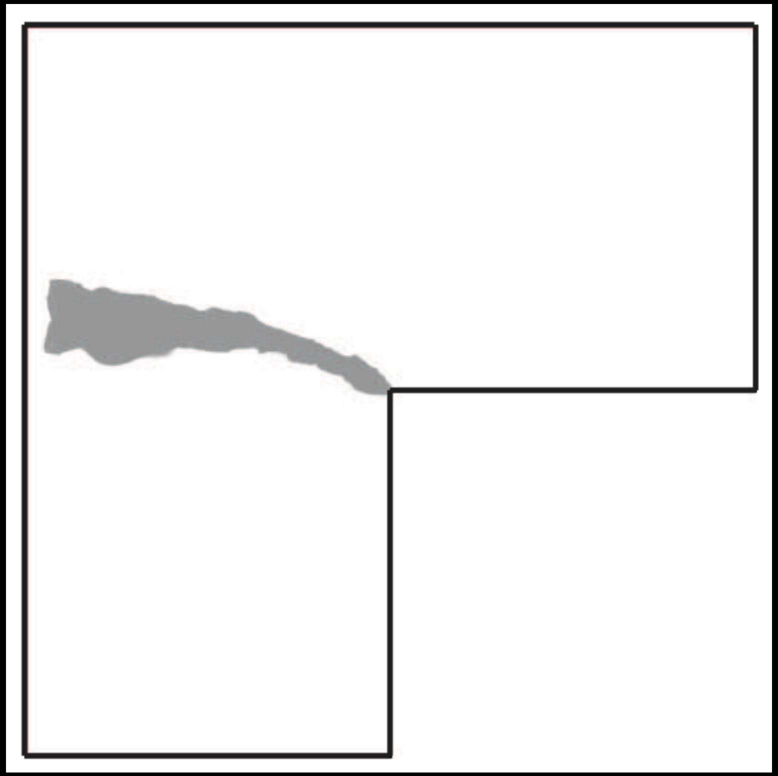
$$\alpha(d) = 2d - d^2$$

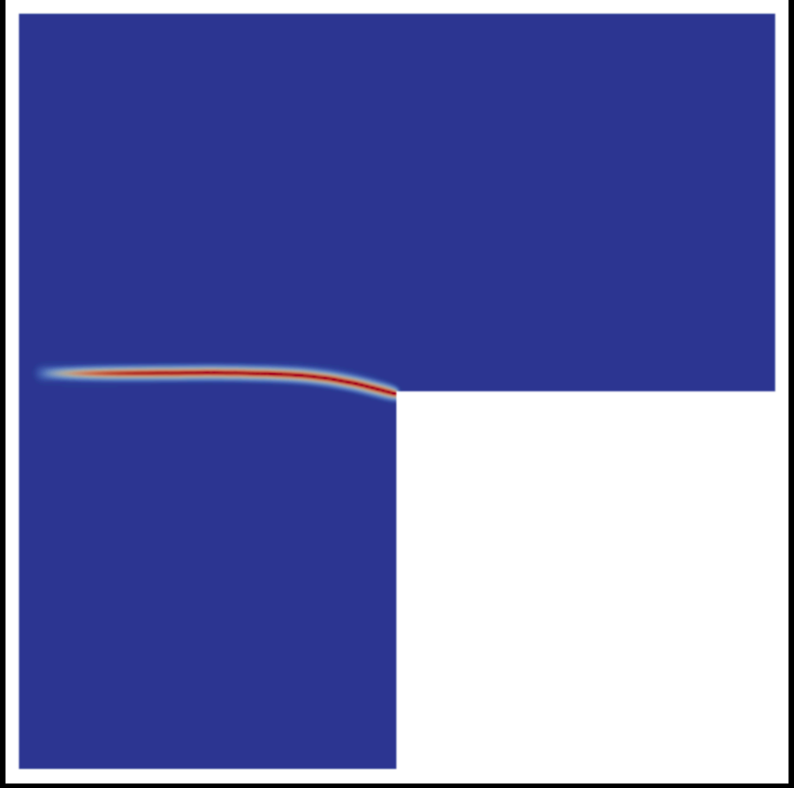


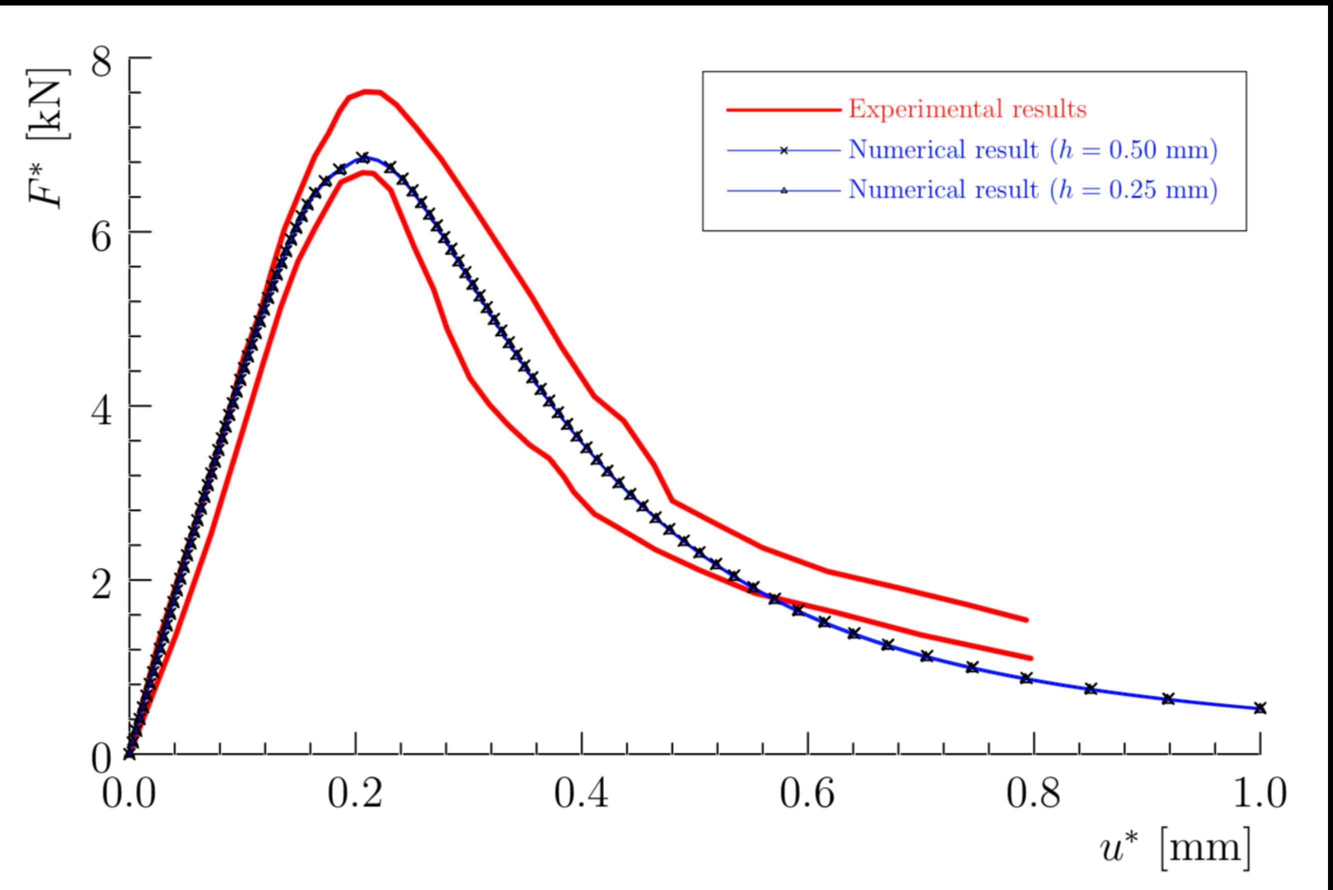
J. Y. Wu. A unified phase-field theory for the mechanics of damage and quasi-brittle failure. *JMPS*, 2017.
J. Y. Wu and V. P. Nguyen. A length scale insensitive phase-field damage model for brittle fracture. *JMPS*, 2018.

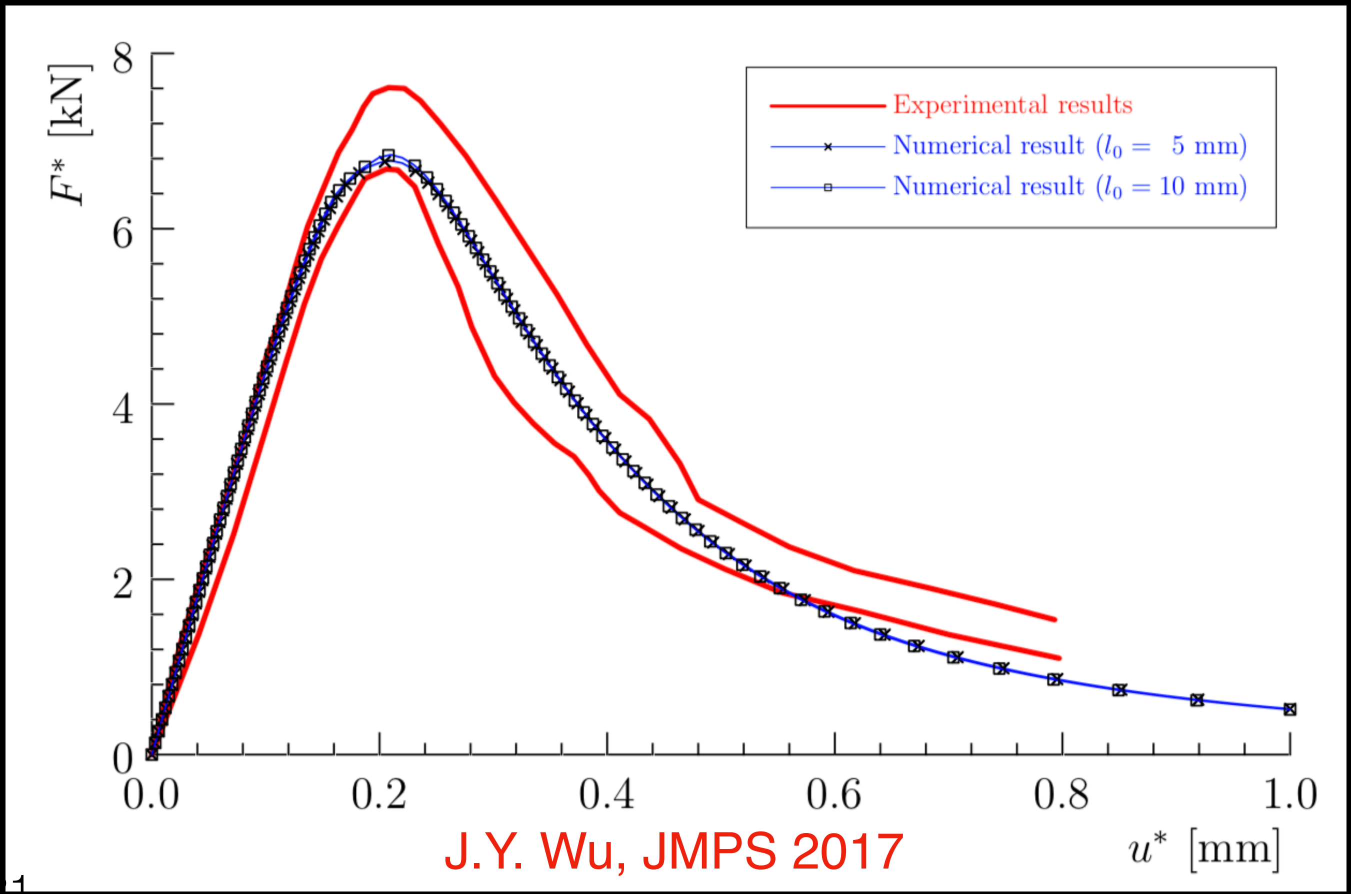
2D quasi-brittle fracture





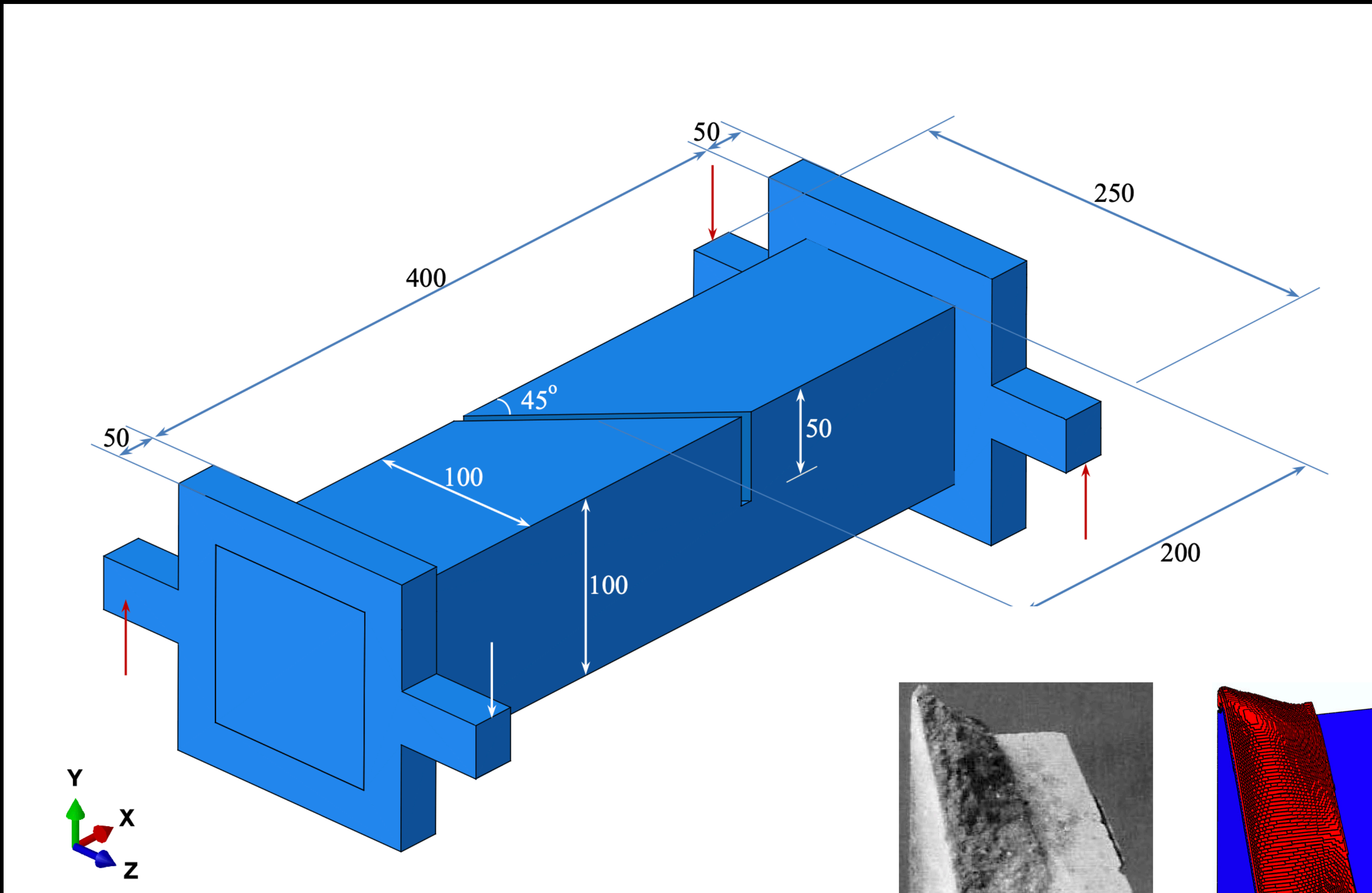


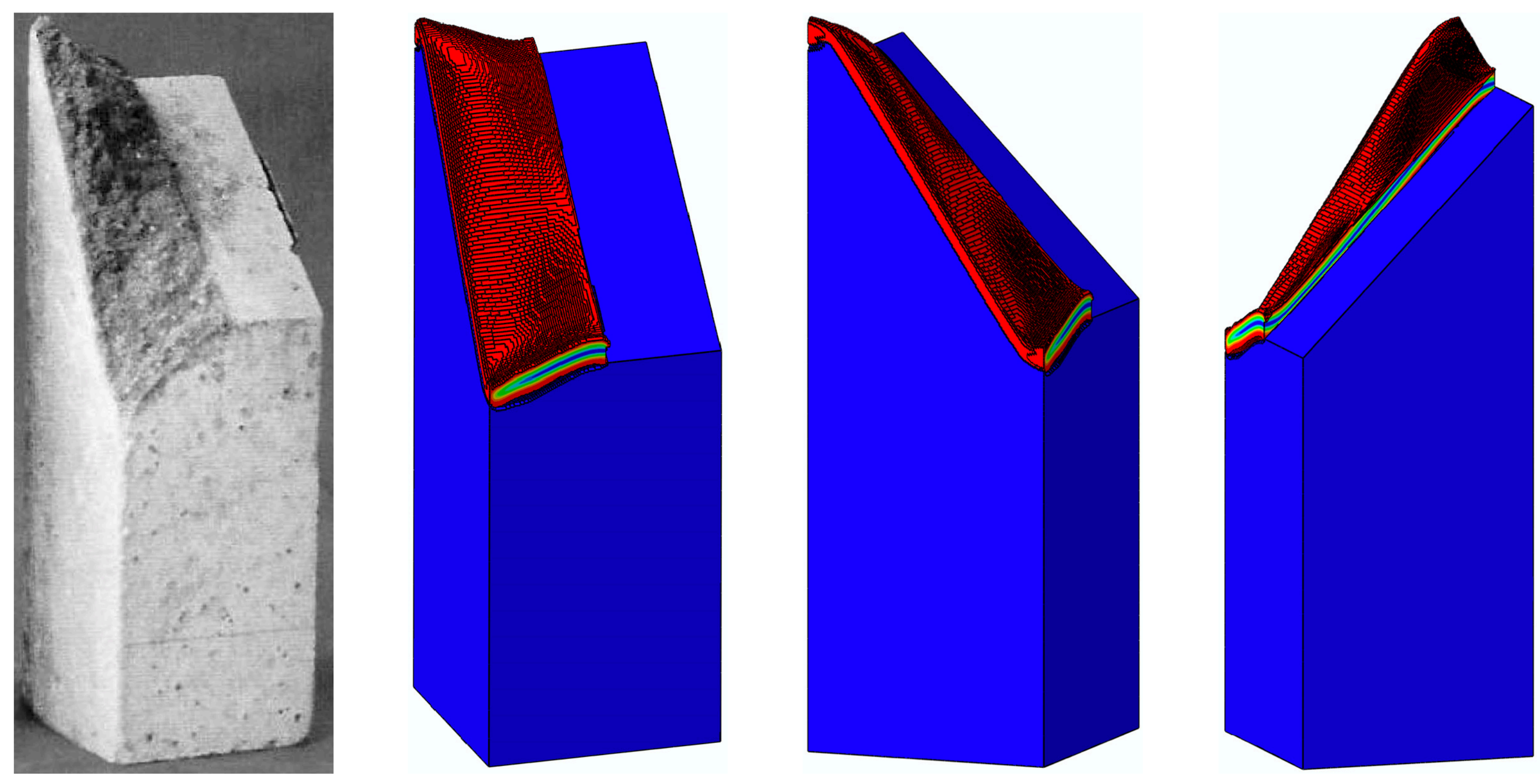




J.Y. Wu, JMPS 2017

3D quasi-brittle fracture

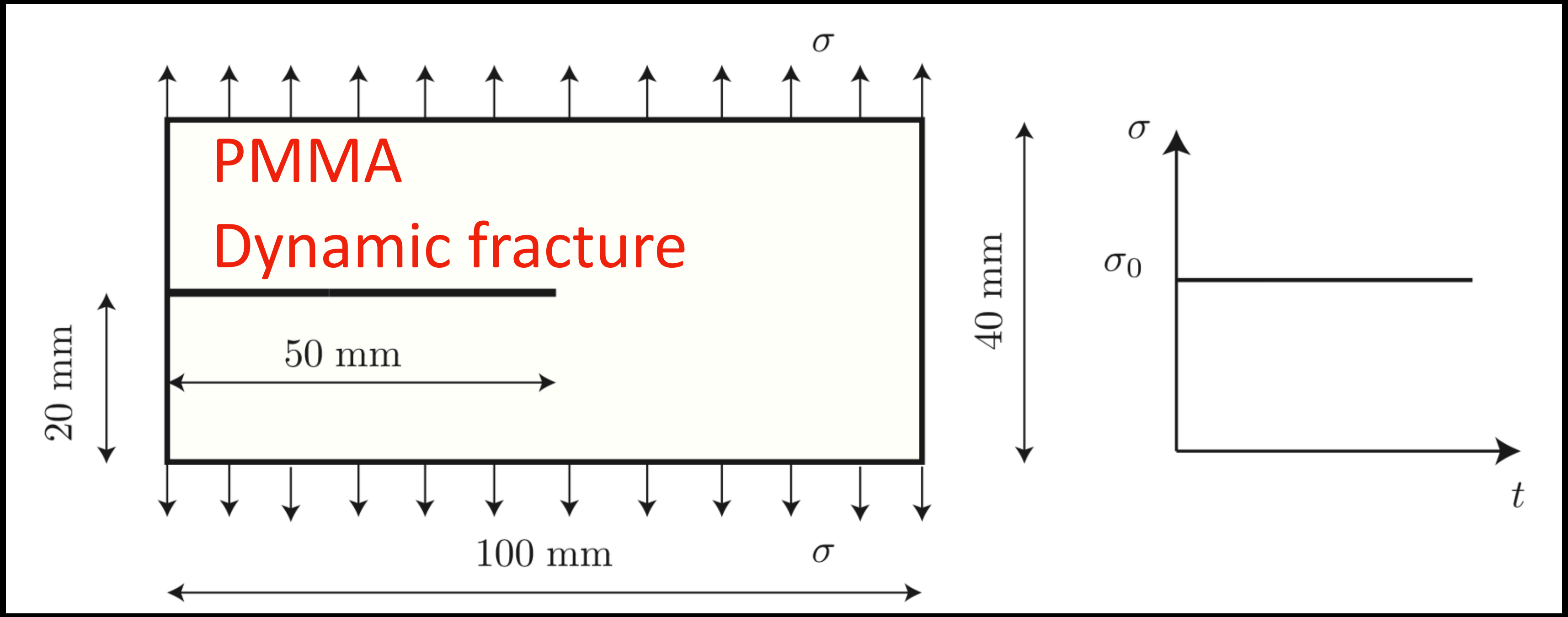


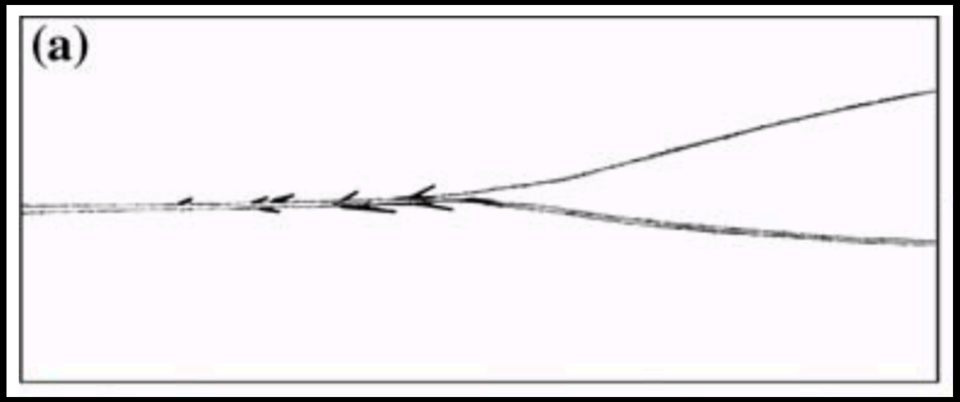


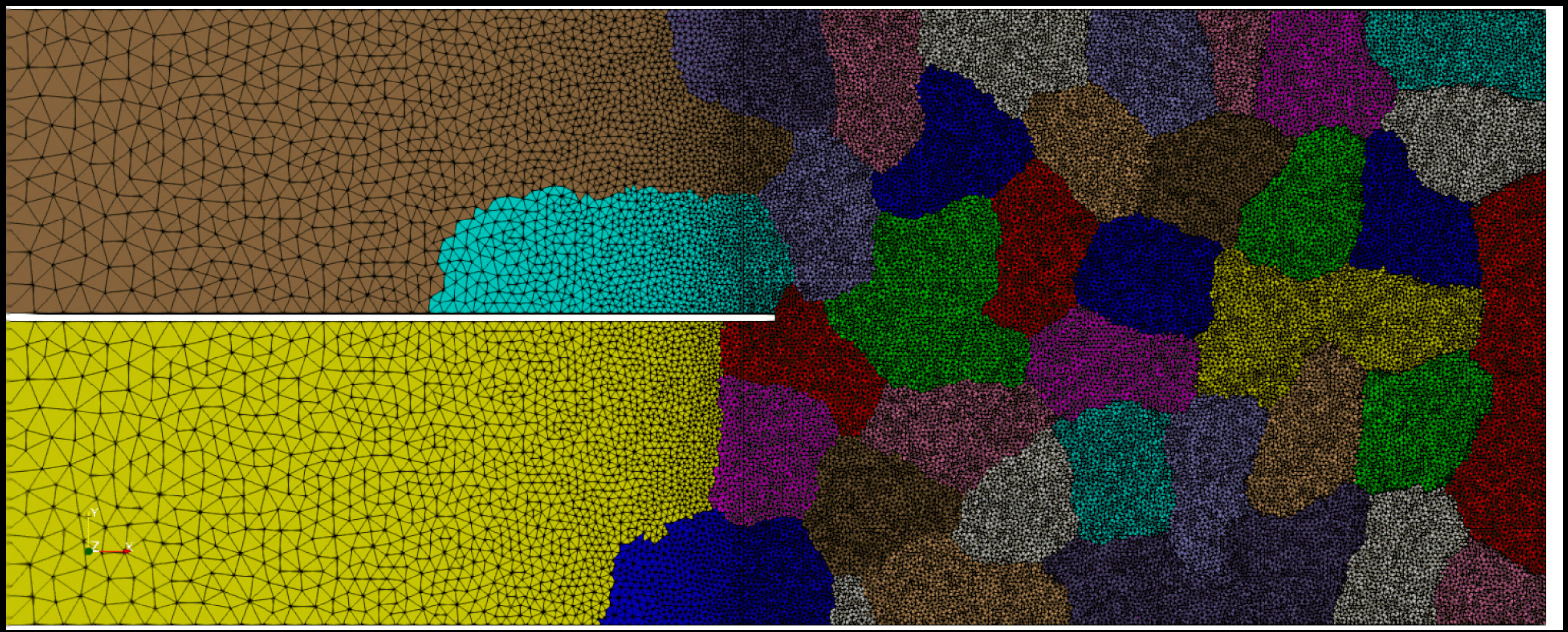
(a) test (b) view 1 (c) view 2 (d) view 3

J.Y. Wu, Y. Huang, H. Zhou,V.P .Nguyen, CMAME 2020

Dynamic brittle fracture:

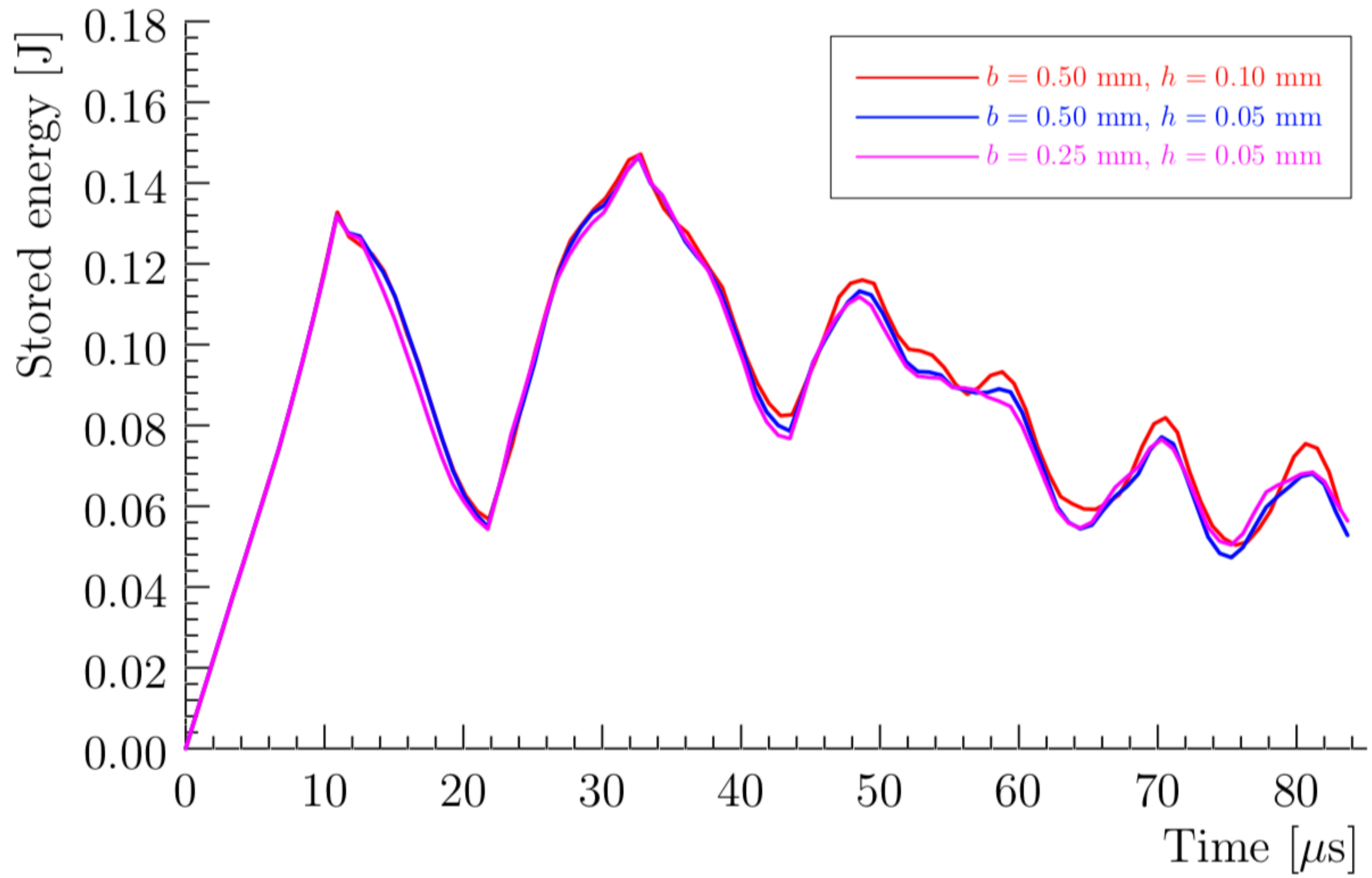






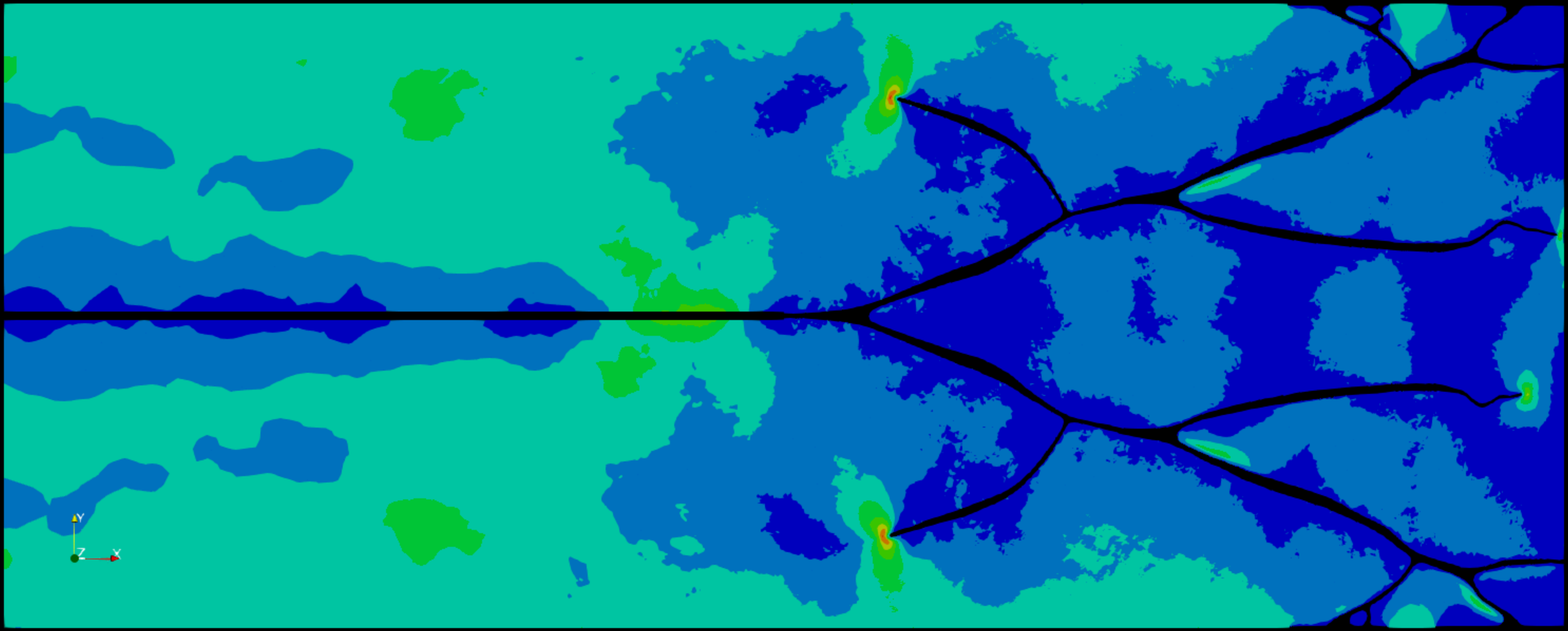
M. J. Borden, C. V. Verhoosel, M. A. Scott, T. J. Hughes, and C. M. Landis.CMAME, 2012.
V.P .Nguyen and J.Y. Wu CMAME 2018
T.K. Mandal. V.P. Nguyen, J.Y. Wu, EFM, 2020

Dynamic brittle fracture:



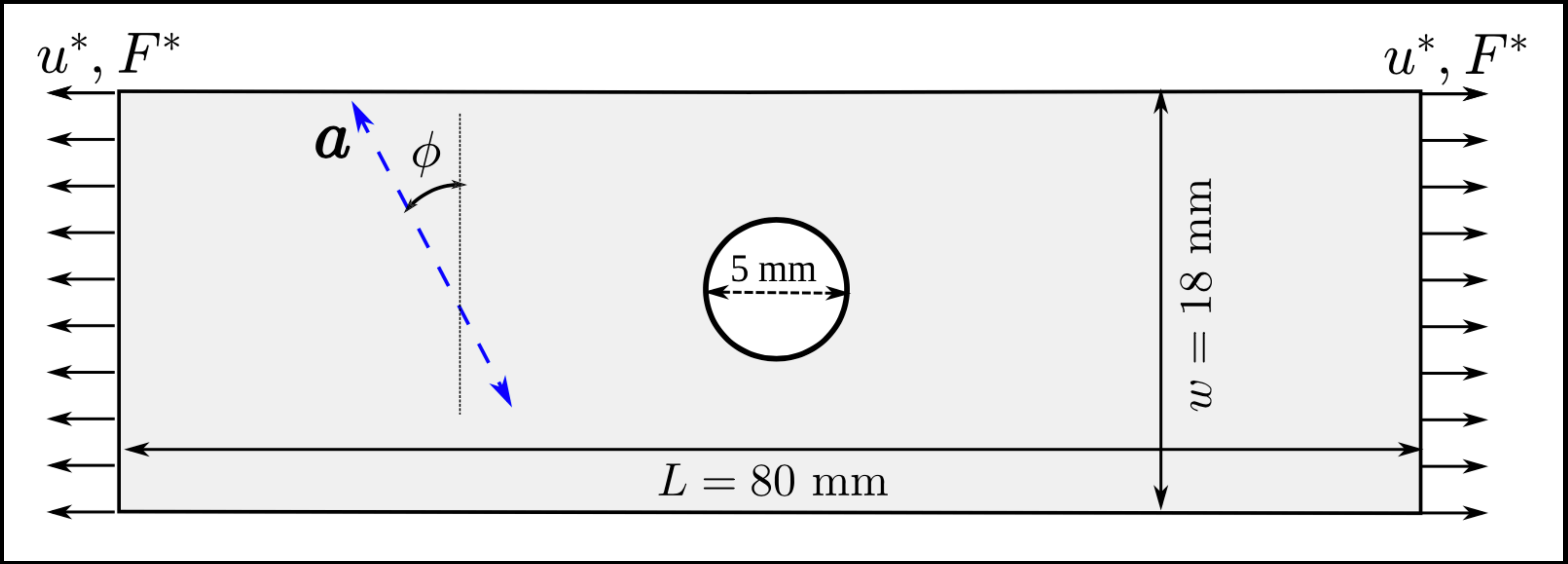
(a) Stored strain energy

Dynamic brittle fracture:



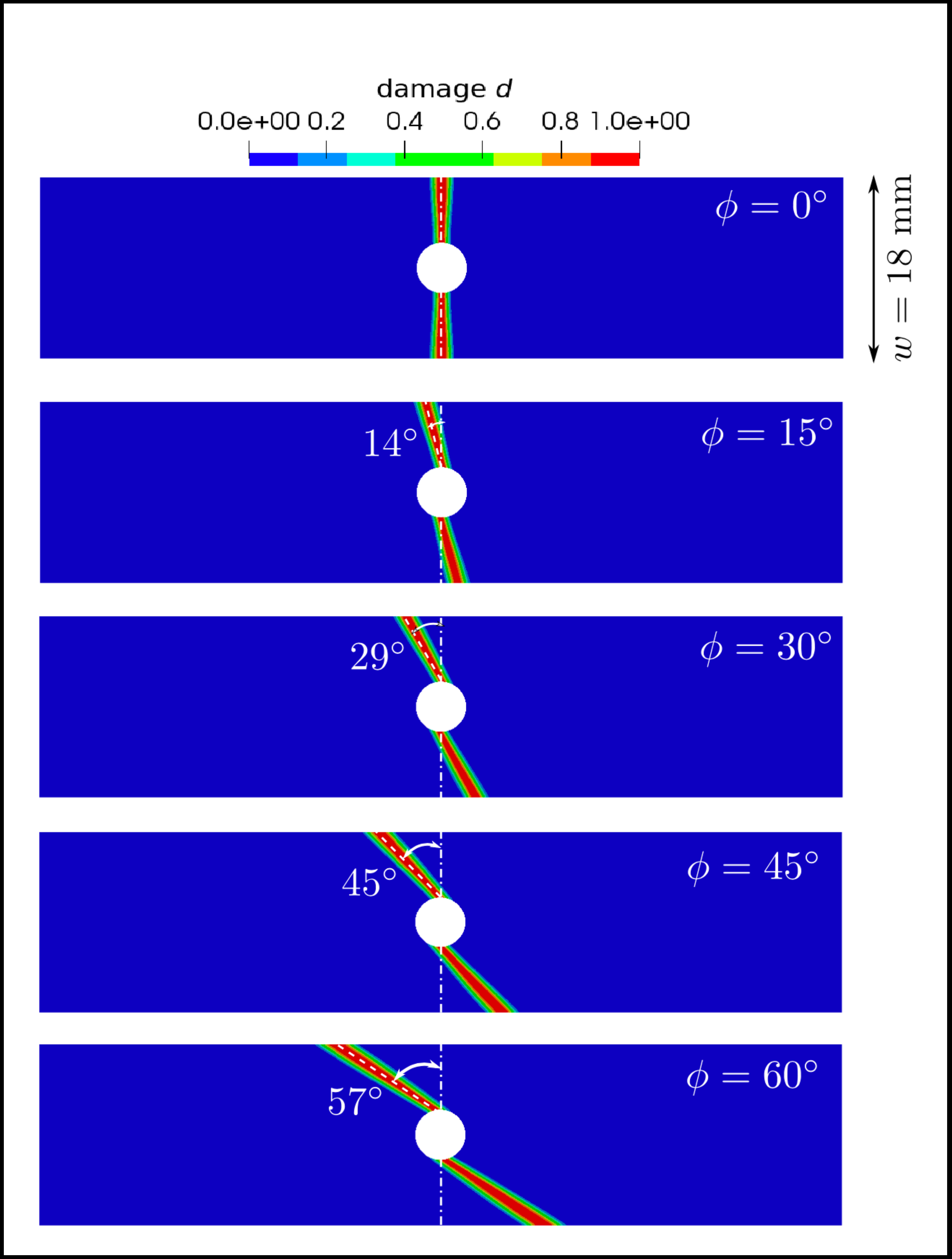
Multiple branching events

Fracture of fibre-reinforced composites



Modniks J, Sprniņš E, Andersons J, Becker W. Analysis of the effect of a stress raiser on the strength of a UD flax/epoxy composite in off-axis tension. Journal of Composite Materials 2015.

Fracture of fibre-reinforced composites



crack follows the fibre

Fracture of fibre-reinforced composites

$$\int_{\Gamma} G_c \mathrm{d}A \approx \int_{\Omega} G_c \gamma(d; \nabla d) \mathrm{d}A$$

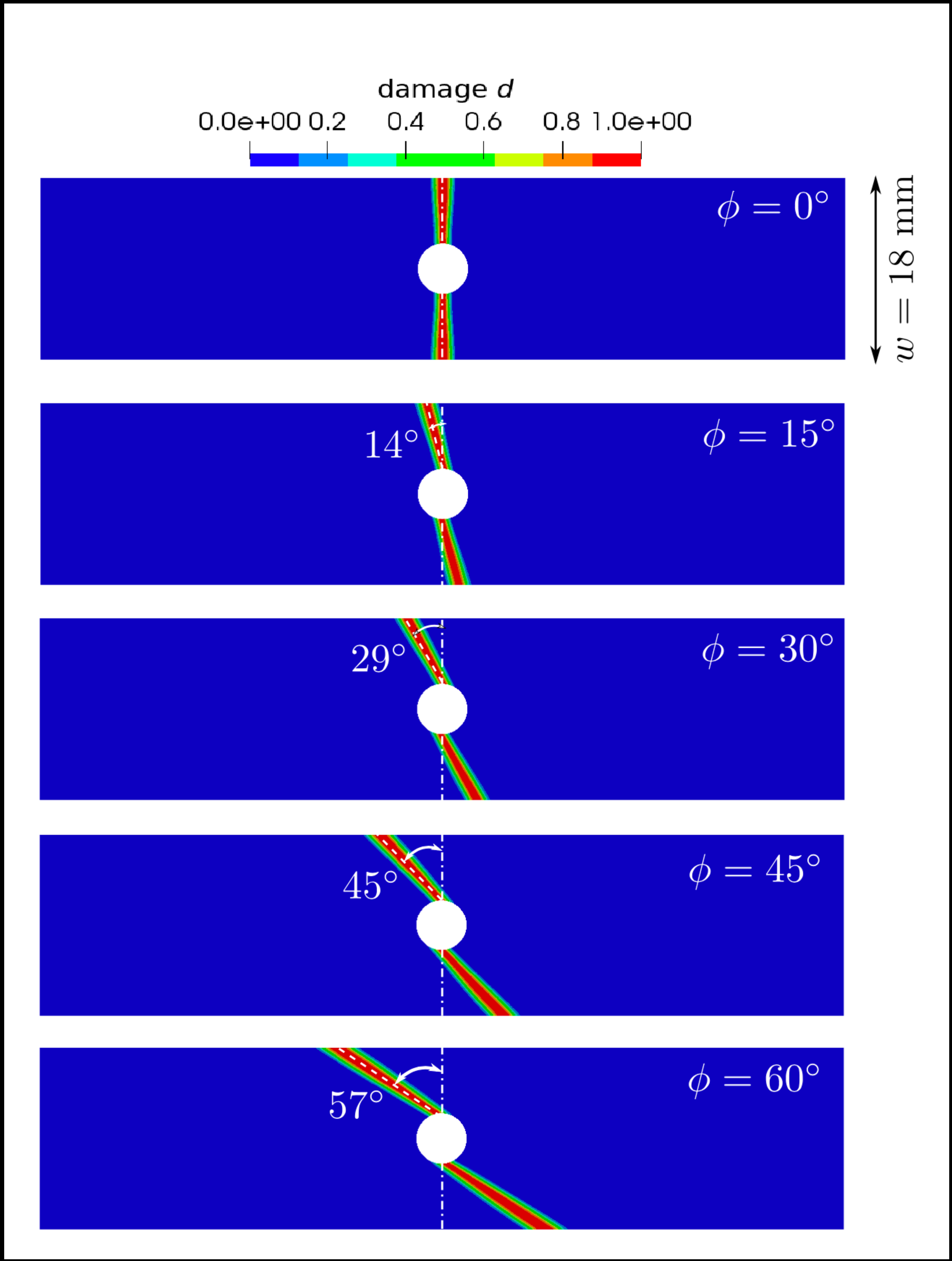
$$\gamma(d; \nabla d) = \frac{1}{c_\alpha}\left[\frac{1}{b}\alpha(d) + b\left|\nabla d\right|^2\right]$$

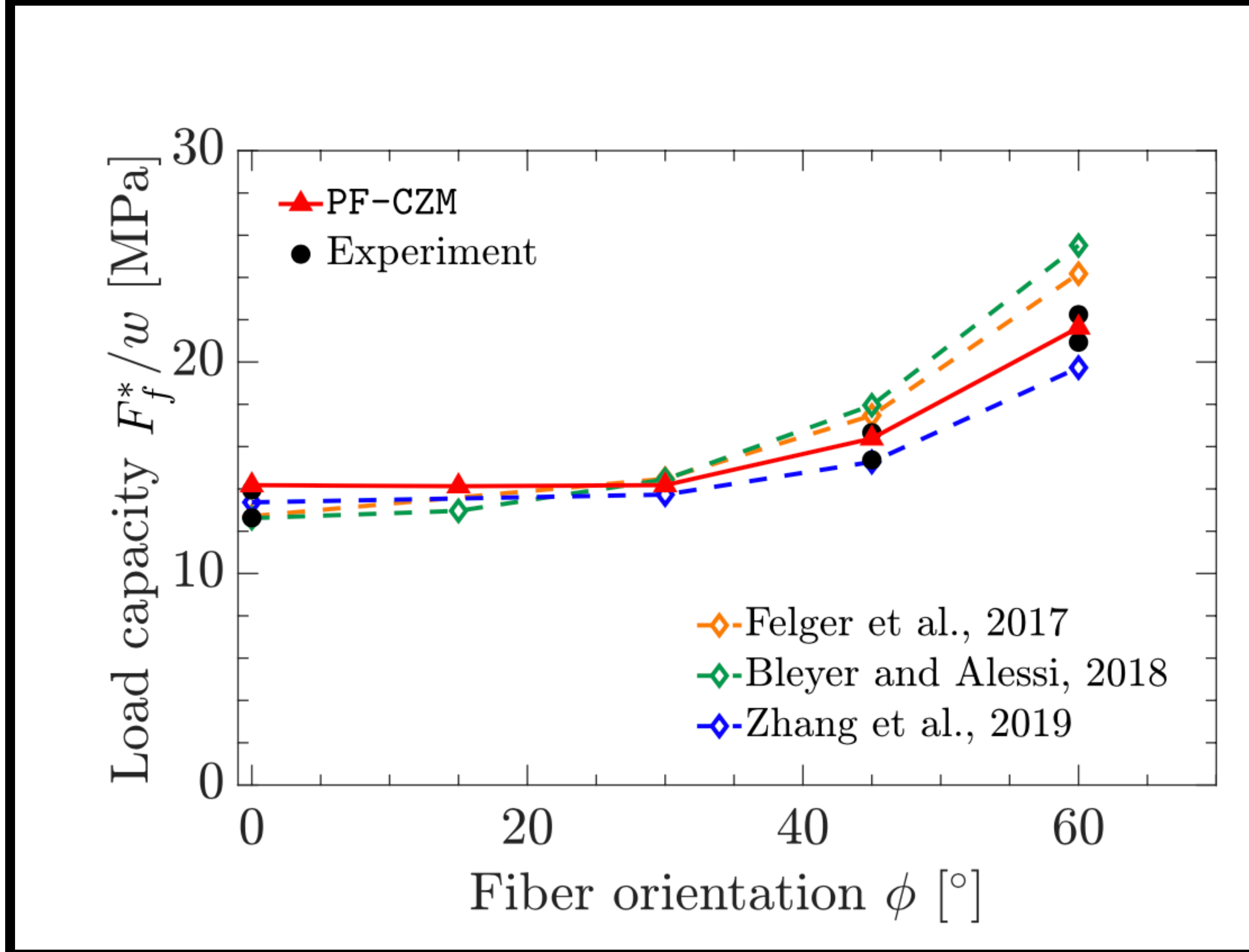
$$\gamma(d; \nabla d) = \frac{1}{c_\alpha}\left[\frac{1}{b}\alpha(d) + b\nabla d \cdot \boldsymbol{A} \cdot \nabla d\right]$$

$$\boldsymbol{A} = \mathbf{1} + \sum_p \xi_p \boldsymbol{a}^p \otimes \boldsymbol{a}^p$$

S. Teichtmeister, D. Kienle, F. Aldakheel, and M.-A. Keip. International Journal of Non-Linear Mechanics, 2017
T.K. Mandal. V.P. Nguyen, J.Y. Wu, IJMS, 2020

Fracture of fibre-reinforced composites





T.K. Mandal. V.P. Nguyen, J.Y. Wu, IJMS, 2020

Thermally induced fracture: thermo-mechanical-damage formulation

Governing equations

$$\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{b} = \boldsymbol{0} \qquad \text{in } \Omega$$

$$g'(d)\psi_0(\boldsymbol{\epsilon}) + G_c\, \delta_d \gamma = 0 \qquad \text{in } \mathcal{B}$$

$$\boldsymbol{\sigma} \cdot \boldsymbol{n} = \boldsymbol{t} \qquad \text{on } \partial\Omega_t$$

$$\frac{\partial \gamma}{\partial \nabla d} \cdot \boldsymbol{n}_{\mathcal{B}} = 0 \qquad \text{on } \partial\mathcal{B}$$

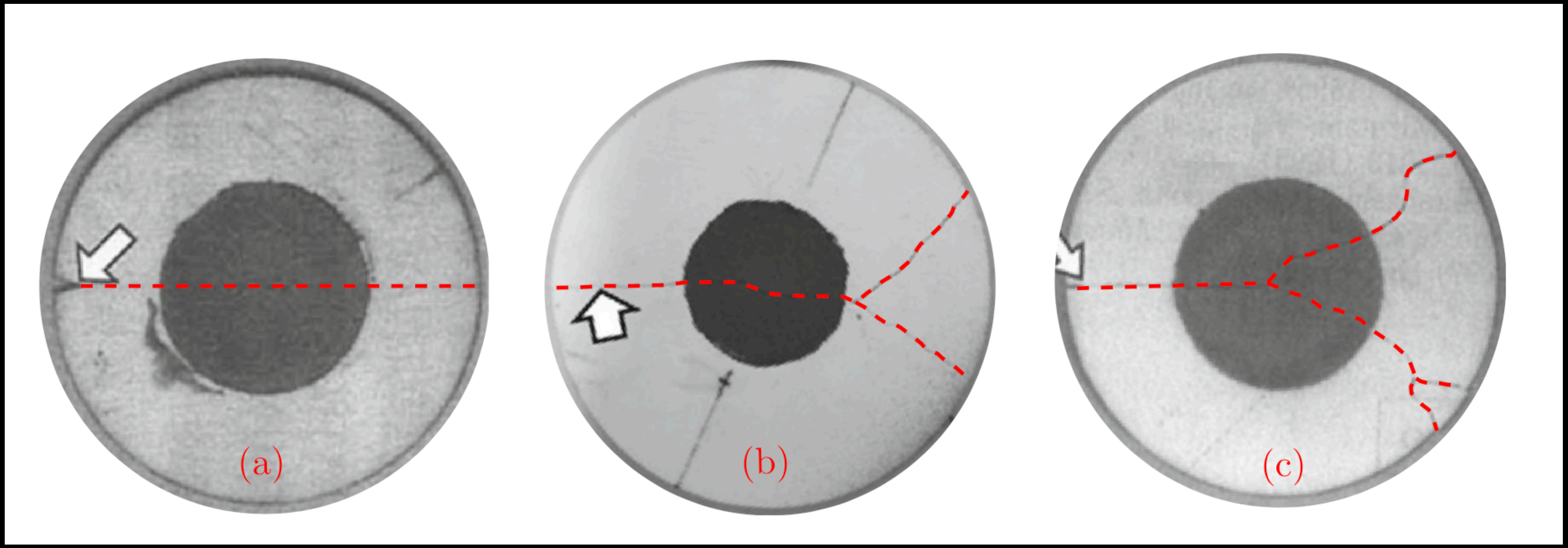
$$\rho c \dot{\theta} + \nabla \cdot \boldsymbol{J} = \gamma^*$$

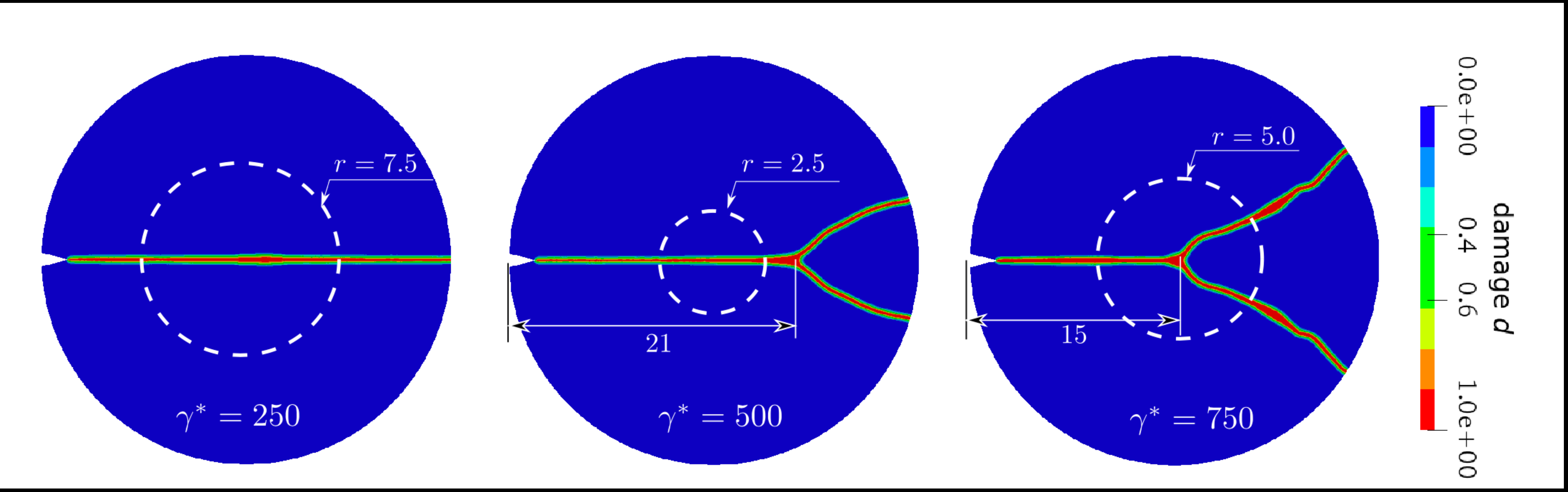
$$\boldsymbol{\sigma} = g(d)\mathbb{E}_0 : (\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta)$$

$$\boldsymbol{\epsilon}_\theta = \alpha(\theta - \theta_0)\mathbf{1}$$

$$\boldsymbol{J} = -k\nabla\theta$$

Thermally induced fracture:





S. Honda, et al. Journal of the Ceramic Society of Japan, 2002
T.K. Mandal. V.P. Nguyen et al.,, 2020

Conclusions 1

$$\Psi(\boldsymbol{u}, d) = \int_\Omega g(d)\psi_0(\boldsymbol{\epsilon}(\boldsymbol{u}))\mathrm{d}V + \int_\Omega G_c\gamma(d; \nabla d)\mathrm{d}V - \mathcal{P}$$

strain energy

surface/fracture energy

Works for **brittle/cohesive fracture**

Works for **static/dynamic/thermo fracture**

Works for **both 2D/3D: same implementation**

Few parameters: elastic properties, tensile strength & fracture energy

Very robust

Conclusions 2

$$\Psi(\boldsymbol{u}, d) = \int_\Omega g(d)\psi_0(\boldsymbol{\epsilon}(\boldsymbol{u}))\mathrm{d}V + \int_\Omega G_c \gamma(d; \nabla d)\mathrm{d}V - \mathcal{P}$$

strain energy

surface/fracture energy

extended for ductile fracture

extended for fatigue

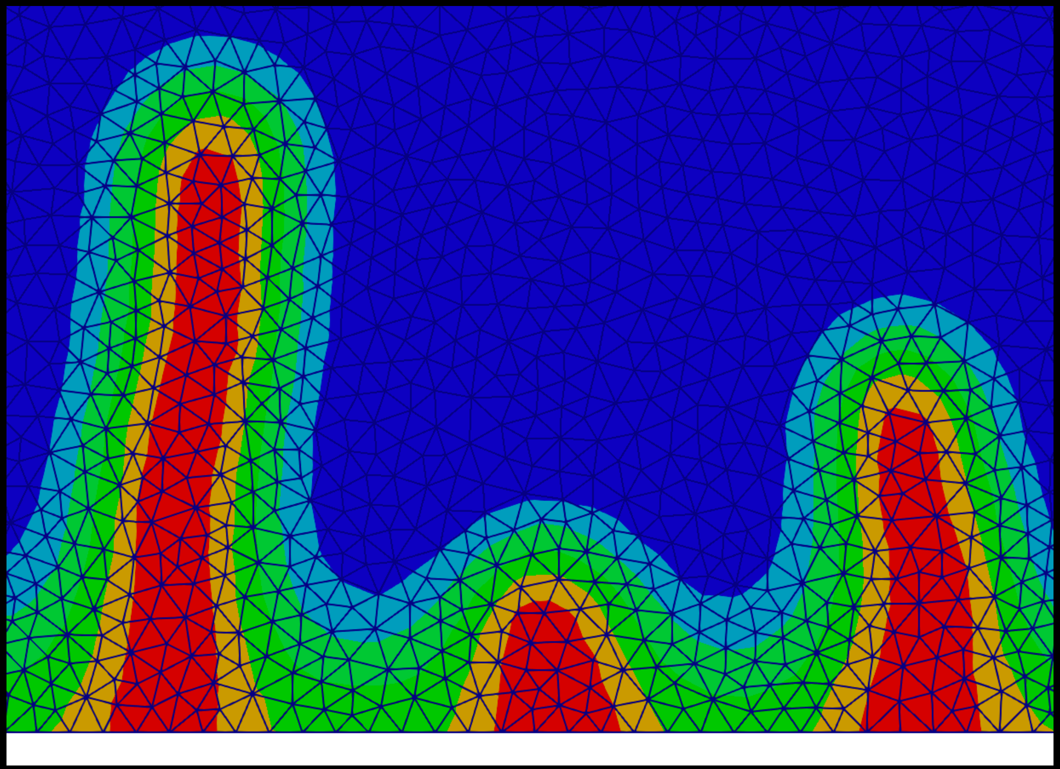
extended for hydrogen assisted cracking

Conclusions 3

$$\Psi(\boldsymbol{u}, d) = \int_\Omega g(d)\psi_0(\boldsymbol{\epsilon}(\boldsymbol{u}))\mathrm{d}V + \int_\Omega G_c\gamma(d;\nabla d)\mathrm{d}V - \mathcal{P}$$

strain energy

surface/fracture energy



too many elements
across the damage band

The end

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